The Multi-Agent Rendezvous Problem

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What is the Multi-Agent Rendezvous Problem?

Convergence in the synchronous case

Achieving emergent behavior via local control – the key idea

Target points

Trapping

Pattern generation

Convergence in the asynchronous case

Asynchronous behavior analyzed using a synchronous but non-deterministic discrete time system.

The Problem

Consider a set of \( n \) mobile autonomous agents which can all move in the plane.

Each agent is able to continuously sense the relative positions of all other agents in its "sensing region" where by agent \( i \)'s a sensing region is meant a closed disk of radius \( r \) centered at agent \( i \)'s current position.

Problem: Devise local control strategies, one for each agent, which without active communication between agents, cause all members of the group to eventually rendezvous at a single unspecified point.


Stop and Go Maneuvers

A stop and go maneuver takes place within a time interval consisting of two successive subintervals:

Each agent is stationary during each sensing period.

During a maneuvering period, an agent moves from its current rest position to its next "way-point" and again comes to rest.

Successive way-points for each agent are constrained to be within \( r \) units of each other.

\( \tau_M \) is an upper bound on the time it takes for agent \( i \) to move from one way-point to the next.

Details of maneuvering to way-points not considered ……or modelled.
Stop and Go Maneuvers

A stop and go maneuver takes place within a time interval consisting of two successive subintervals:

\[ T_L, \quad T_M \]

\[ ^{k\text{th}} \text{sensing period} \quad ^{k\text{th}} \text{maneuvering period} \]

Synchronous Operation

\( T_M \) is the same for all agents.

All agents use a common clock.

Sensing periods for all agents occur at the same time.

Registered neighbors of agent \( i \) at beginning of \( k \)th maneuvering period are those agents within agent \( i \)'s sensing region during its \( k \)th sensing period.

Registered neighbors of agent \( i \) at time \( t_k \) are those agents at distances no greater than \( r \) units from agent \( i \) at time \( t_k \).

Registered neighbors is a symmetric relation: If \( A \) is a registered neighbor of \( B \) at time \( t_k \), then \( B \) is a registered neighbor of \( A \) at time \( t_k \).

Registered neighbors is not a transitive relation: It is not necessarily true that if at time \( t_k \), \( A \) is a registered neighbor of \( B \) and if \( B \) is a registered neighbor of \( C \), then \( A \) is a neighbor of \( C \).

However, registered neighbors is weakly transitive: If \( A \) is at the same position as \( B \) at time \( t_k \) and if \( C \) is a neighbor of \( B \) at time \( t_k \), then \( C \) is a neighbor of \( A \) at time \( t_k \).

An emergent behavior: Weak transitivity has a global implication – If at time \( t_k \), any one agent has rendezvoused locally with each of its neighbors and if the associated neighbor graph is connected, then the entire group has rendezvoused! If at time \( t_k \), any one agent has rendezvoused locally with each of its neighbors and if the associated neighbor graph is connected, then the entire group has rendezvoused!}

Emergent (global) behavior via local control – The key idea

By the local convex hull (at time \( t \)) of agent \( i \) is meant the convex hull [at time \( t \)] of its position and the positions of its registered neighbors.

By the global convex hull (at time \( t \)) of the group is meant the convex hull [at time \( t \)] of the positions of all \( n \) agents.

Local rules:
1. Each agent remain within its local convex hull.
2. Each agent at a corner of its local convex hull moves inside or at least away from all corners...if it can.

Since the local convex hulls are subsets of the global convex hull, the local rules imply that the global convex hull can't grow as agents move. Does not mean local convex hulls get smaller!

Since each corner of the global convex hull must be a corner of some local convex hull, the second local rule cause all agents at corners of the global convex hull to move away from corners.

Since the diameter of a convex hull is always the distance between two of its corners, the local rules cause the diameter of the global convex hull to decrease.

Emergent Behavior

Global Goal: Shrink the global convex hull by local control

Local rule: Each agent moves “inside” of its local convex hull

Some local convex hulls get smaller
Some local convex hulls can get larger

But the global convex hull cannot get larger and may get smaller.
Emergent (global) behavior via local control – The key idea

By the local convex hull (at time t) of an agent is meant the convex hull [at time t] of the positions of its registered neighbors.

By the global convex hull (at time t) of the group is meant the convex hull [at time t] of the positions of all n agents.

Local rules: 1. Each agent remain within its local convex hull.
2. Each agent at a corner of its local convex hull moves inside or at least away from all corners if it can.

Since the local convex hulls are subsets of the global convex hull, the local rules imply that the global convex hull cannot grow as agents move.

Since each corner of the global convex hull must be a corner of some local convex hull, the second local rule cause all agents at corners of the global convex hull to move away from corners.

Since the diameter of a convex hull is always the distance between two of its corners, the local rules cause the diameter of the global convex hull to decrease.

Connectivity is needed for convergence.

A simple graph $G = (V, E)$ with vertex set $V = \{1, 2, \ldots, n\}$ and edge set $E$ defined so that $(i,j) \in E \iff$ agents i and j are registered neighbors.

Let $(G_p : p \in P)$ denote the indexed set of all simple graphs $G_p = (V, E_p)$, with vertex set $V = \{1, 2, \ldots, n\}$ and partially ordered by the relation $G_p \subset G_q \iff E_p \subset E_q$.

Let $G_{pd}$ denote the graph describing the $n$ agents’ neighbor relationships during $k$th maneuvering period.

Note that if the $n$ agents are cooperating, then the sequence of graphs $G_{p10}, G_{p20}, \ldots$ satisfy the ascending chain $G_{p10} \subset G_{p20} \subset \ldots$

Because $(G_p : p \in P)$ contains only a finite number of distinct graphs, there must be a finite $k^*$ beyond which $G_{pkd}$ does not change.

Thus $G_{p(k)} = G^*$, $k \geq k^*$

where $G^* = \left\{ V, \bigcup_{k \geq 1} E_{p(k)} \right\}$

In other words, if the $n$ agents are cooperating, there is a finite $k$ beyond which each agent’s set of registered neighbors does not change. Moreover, if the neighbor graph is initially connected, then the final neighbor graph is connected as well.
A pair of agents which are registered neighbors during the kth maneuvering period satisfy the pairwise motion constraint if the points to which they respectively move at the end of the kth maneuvering period are within a closed disk of diameter r centered at the mean of their positions at the beginning of the kth maneuvering period.

Either contains just agent A or is strictly convex with nonempty interior

\[
x(t) = \text{position of agent } i \text{ at time } t
\]

\[
\{i_1, i_2, \ldots, i_{m_i(k)}\} = \text{set of indices of registered neighbors of agent } i \text{ during kth maneuvering period.}
\]

Agent i's kth way-point = \(x_i(t_{k-1}) + u_{m_i(k)}(z_1, z_2, \ldots, z_{m_i(k)})\)

Relative position of agent i's jth registered neighbor:

\(z_j = x_j(t_{k-1}) - x_i(t_{k-1}), \quad j \in \{1, 2, \ldots, m_i(k)\}\)

\(u_m : \mathbb{D}^m \to \mathbb{D}, \quad m \in \{1, 2, \ldots, n - 1\}\)

\(\mathbb{D} = \text{disk of radius } r \text{ centered at the origin}\)

Way-point Update Rules

\(u_m : \mathbb{D}^m \to \mathbb{D}\)

0. \(u_0 = 0\)

1. \(u_m(z_1, z_2, \ldots, z_m) \in (0, z_1, z_2, \ldots, z_m) = \text{convex hull of } 0, z_1, z_2, \ldots, z_m\)

2. \(u_m(z_1, z_2, \ldots, z_m) \in C(z_1, z_2, \ldots, z_m) = \text{constraint set of } z_1, z_2, \ldots, z_m\)

3. \(u_m : \mathbb{D}^m \to \mathbb{D} \) is continuous.

4. \(u_m(z_1, z_2, \ldots, z_m) \neq \text{a corner of } (0, z_1, z_2, \ldots, z_m)\)

If all n agents update their positions according to these rules, then the position of each eventually converges to one of \(q \cdot n\) distinct points in the plane, which are separated from each other pair-wise by a distance greater than r.

Moreover if two agents are registered neighbors during any maneuvering period, then both converge to the same point.
A continuous function $\tau : \mathbb{D}^m \to \mathbb{D}$ is a target point if

1. $(z_1, z_2, \ldots, z_m) \in \{0, z_1, z_2, \ldots, z_m\}$

2. The line segment from 0 to $(z_1, z_2, \ldots, z_m)$ which lies within $c(z_1, z_2, \ldots, z_m)$ has positive length whenever 0 is a corner of $(0, z_1, z_2, \ldots, z_m)$.

$$u_m(z_1, z_2, \ldots, z_m) = g(z_1, z_2, \ldots, z_m) \tau(z_1, z_2, \ldots, z_m)$$

$g : \mathbb{D}^m \to \mathbb{R}$ is any continuous positive definite function satisfying

$$g(z_1, z_2, \ldots, z_m) < \max_{\mu} \{ \mu \tau(z_1, z_2, \ldots, z_m) \in c(z_1, z_2, \ldots, z_m)\}$$

The problem then is to define target points which depend continuously on neighbor positions and in addition which enable those agents at corners of their local convex hulls to move away from such corners in the direction of their target points while remaining in their constraint sets.

Nor is the average of the points $0, z_1, z_2, \ldots, z_m$, namely

$$\frac{1}{m+1} \sum_{i=0}^{m} z_i$$

because it fails to satisfy the condition.
If 0 is a corner of local convex hull, the segment of the line from 0 to target which lines in constraint set must have positive length. Target must be continuous function of neighbor positions.

Target is centroid of intersection disks of radius r centered at neighbor’s positions.

Pattern Generation

Open Problem: Given an initial distribution of the positions of a group of movable agents together with the positions of specific non-movable agents, determine the positions of all the points at which agents rendezvous.

Asynchronous Operation

As in the synchronous case, each agent still executes a sequence of consecutive stop and go maneuvers. For agent i:

\[ \tau_D \leq \tau_M. \]

kth sensing period
\( \tau_D \)

kth maneuvering period
\( \tau_M \)

Agent i’s event times:
\( t_0, t_1, \ldots \)

Not synchronized with other agents’ event times

It is assumed that agent i reaches its kth waypoint within its kth maneuvering period and then comes to rest.

Since agents don’t move during sensing periods, agent i’s kth waypoint is the same as its position at its \((k+1)\)th event time.

We need to define what is meant by a registered neighbor of agent i.
Asynchronous Operation

As in the synchronous case, each agent still executes a sequence of consecutive stop and go maneuvers. For agent $i$:

- $k$th sensing period
- $k$th maneuvering period

A design parameter called a sensing time $\tau_s > 0$ is chosen to satisfy

$$\tau_s \leq \frac{1}{2}(\tau_D - \tau_M), \quad i \in \{1, 2, \ldots, n\}$$

Registered neighbors of agent $i$ during its $k$th maneuvering period are those agents which during agent $i$'s $k$th sensing period are stationary for at least $\tau_s$ seconds at positions within agent $i$'s sensing region.

For a given registered neighbor $j$, there may be more than one distinct interval of length at least $\tau_s$ in sensing period $k$ during which $j$ is fixed.

Registered position of neighbor $j$ is taken as right end point of last of these

For neighbor symmetry, need to make sure that this “registration interval” also lies within one of agent $j$'s sensing periods.

Asynchronous Agent System

As in the synchronous case, each agent still executes a sequence of consecutive stop and go maneuvers. For agent $i$:

- $k$th sensing period
- $k$th maneuvering period

Since agents don’t move during sensing periods, agent $i$'s $k$th waypoint is the same as its position at its $(k + 1)$st event time.

$$x_i(t_{k+1}) - x_i(t_k) + u_{max}(x_i(t^*_1) - x_i(t_k), \ldots, x_i(t^*_m) - x_i(t_k))$$

$u_{max}$ same as before

$\{l_1, l_2, \ldots, l_m\} =$ labels of agent $i$'s registered neighbors on maneuvering period $k$

One equation of this type for each agent
A Common Time Scale

It is possible to characterize system behavior with respect to a common time scale \( T \) whose elements are re-labeled versions of all distinct event times of all agents.

\[ \{t_{ik} : i \in \{1, 2, \ldots, n\}, \ k \geq 1 \} \]

In particular, \( T \) is the ordered set of event times whose elements are labeled \( t_1, t_2, \ldots \) in such a way so that \( t_{i+1} > t_i \).

Let \( \sigma(k) \) denote that value of \( s \) for which \( t_{ik} = \tau k \).

Extend the domain of definition of agent \( i \)'s position and registered neighbors from agent \( i \)'s event time set to the set \( \{t_s : s \geq 1\} \) by stipulating that for values of \( t_s \) between two successive event times of agent \( i \), say between \( t_{ik} \) and \( t_{i(k+1)} \), agent \( i \)'s position and registered neighbors are respectively the same as its position and registered neighbors at time \( t_0 \).

\[
\mathbb{G} = \{\mathbb{G}_p : p \in \mathcal{P}\}
\]

with vertex set \( \mathbb{V} = \{1, 2, \ldots, n\} \) and partially ordered by the relation \( \mathbb{G}_p \subseteq \mathbb{G}_q \Leftrightarrow E_p \subseteq E_q \).

Let \( \mathbb{G}_p(s) \) denote the graph describing the \( n \) agents' neighbor relationships at event time \( t_s \).

Note that if the \( n \) agents are cooperating, then the sequence of graphs \( \mathbb{G}_p(1), \mathbb{G}_p(2), \ldots \) satisfy the ascending chain \( \mathbb{G}_p(1) \subset \mathbb{G}_p(2) \subset \ldots \).

Because \( \mathbb{G}_p(p \in \mathcal{P}) \) contains only a finite number of distinct graphs, there must be a finite \( s^* \) beyond which \( \mathbb{G}_p(s) \) does not change.

Thus \( \mathbb{G}_p(s) = \mathbb{G}_p(s^*) \ for \ s \geq s^* \)

where \( \mathbb{G}^* = \{\mathbb{V}_i \bigcup \mathbb{E}_{\mathbb{G}_p(s)}\} \)

A directed graph \( \mathbb{G} = (\mathbb{V}, \mathbb{E}) \) with vertex set \( \mathbb{V} = \{1, 2, \ldots, n\} \) and edge set \( \mathbb{E} \) defined so that \( (i, j) \in \mathbb{E} \) if \( j \) is a registered neighbor of agent \( i \).

Let \( \{G_p, p \in \mathcal{P}\} \) denote the indexed set of all directed graphs \( \mathbb{G}_p = (\mathbb{V}, \mathbb{E}_p) \) with vertex set \( \mathbb{V} = \{1, 2, \ldots, n\} \) and partially ordered by the relation \( \mathbb{G}_p \subseteq \mathbb{G}_q \iff E_p \subseteq E_q \).

Let \( \mathbb{G}_{\text{reg}} \) denote the graph describing the \( n \) agents' neighbor relationships at event time \( t_0 \).

Note that if the \( n \) agents are cooperating, then the sequence of graphs \( \mathbb{G}_p(1), \mathbb{G}_p(2), \ldots \) satisfy the ascending chain \( \mathbb{G}_p(1) \subset \mathbb{G}_p(2) \subset \ldots \).

Because \( \mathbb{G}_p(p \in \mathcal{P}) \) contains only a finite number of distinct graphs, there must be a finite \( s^* \) beyond which \( \mathbb{G}_p(s) \) does not change.

Thus \( \mathbb{G}_p(s) = \mathbb{G}_p(s^*) \ for \ s \geq s^* \)

where \( \mathbb{G}^* = \{\mathbb{V}_i \bigcup \mathbb{E}_{\mathbb{G}_p(s)}\} \)

Asynchronous Operation (after neighbors stop changing)

It is possible to "simulate" the preceding asynchronous behavior with a synchronous discrete-time dynamical system \( S \) evolving on the ordered set \( T = \{t_s : s \geq s^*\} \).

Define \( \mathcal{S} = \{s', s^* + 1, \ldots\} \) and for \( i \in \{1, 2, \ldots, n\} \)

\[ \sigma(k) = \text{value of } s \text{ for which } t_{ik} = \tau k, \quad k \geq 1 \text{, and} \]

\[ S_i = \mathcal{S} \cap \text{Image } \sigma_i \]

\[ w_i(s) = u_i(s - 1), \quad w_i(s) = u_i(s - 1), \quad w_i(s) = \sum_{j=1}^{m_i} u_j(s - 1) - u_j(s - 1), \quad w_i(s) = \sum_{j=1}^{m_i} u_j(s - 1) - u_j(s - 1), \quad j \in \{1, 2, \ldots, m_i\} \]

where \( j \in \mathcal{N}_i \) set of indices of agent \( i \)'s neighbors.

\[ \text{Neighbor Graph} \]

A directed graph \( \mathbb{G} = (\mathbb{V}, \mathbb{E}) \) with vertex set \( \mathbb{V} = \{1, 2, \ldots, n\} \) and edge set \( \mathbb{E} \) defined so that \( (i, j) \in \mathbb{E} \) if \( j \) is a registered neighbor of agent \( i \).

Let \( \{G_p, p \in \mathcal{P}\} \) denote the indexed set of all directed graphs \( G_p = (V, E_p) \) with vertex set \( V = \{1, 2, \ldots, n\} \) and partially ordered by the relation \( G_p \subseteq G_q \iff E_p \subseteq E_q \).

Let \( G_{\text{reg}} \) denote the graph describing the \( n \) agents' neighbor relationships at event time \( t_0 \).

Note that if the \( n \) agents are cooperating, then the sequence of graphs \( G_p(1), G_p(2), \ldots \) satisfy the ascending chain \( G_p(1) \subset G_p(2) \subset \ldots \).

Because \( \{G_p, p \in \mathcal{P}\} \) contains only a finite number of distinct graphs, there must be a finite \( s^* \) beyond which \( G_p(s) \) does not change.

Thus \( G_p(s) = G_p(s^*) \ for \ s \geq s^* \)

where \( G^* = \{V_i \bigcup \bigcup_{s=1}^{\infty} E_{G_p(s)}\} \)
Concluding Remarks

Using the preceding synchronous, nondeterministic model $\mathcal{S}$, it is possible to establish agent convergence to a common, unspecified rendezvous point, provided the graph representing neighbor relationships is connected at some point during the rendezvousing process.

We have very recently discovered that by more completely utilizing information from the original asynchronous agent process, namely the durations between event times, it is possible to convert $\mathcal{S}$ into a deterministic system.

Whether or not this simplifies the analysis remains to be seen.

The results we’ve described have recently been applied to the problem of positioning mobile sensors within a network so as to save transmission power which maintaining network connectivity.