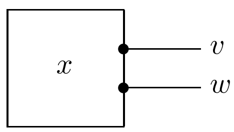
Input to State Stability and Related Notions

E. D. SONTAG
Department of Mathematics, Rutgers University
New Brunswick, NJ 08903
`http://www.math.rutgers.edu/~sontag`

Abstract

The input to state stability (ISS) paradigm provides a way to formulate questions of stability with respect to disturbances, as well as to conceptually unify detectability, input/output stability, minimum-phase behavior, and other systems properties. This series of talks will discuss the main theoretical results concerning ISS and related notions. The proofs of some of the results will be sketched, showing in particular connections to relaxation problems for differential inclusions, converse Lyapunov theorems, and nonsmooth analysis.

The main focus is the "input to state stability" *way of thinking* about nonlinear stability questions. Consider the general "port" picture



where v and w are external signals, x the internal state. We study *conditional (asymptotic) stability* of v, w. There are two desirable, and complementary, features of stability:

- *asymptotic:* "v small implies w small" — where "small" may be interpreted as "$\to 0$ when $t \to +\infty$", "bounded", or via an $\varepsilon - \delta$ definition; and
- *transient:* "overshoot depends on initial state" — with fading effect of $x(0)$.

ISS-type definitions capture these two aspects. The "magnitude" of a signal might be e.g.: the norm: $|w(t)|$, an error: $|w(t) - w_{\text{desired}}(t)|$, or the distance to a set $\mathcal{A}$: $|w(t)|_{\mathcal{A}} = \text{dist}\,(w(t), \mathcal{A})$ (for instance, if $\mathcal{A}$ is a periodic orbit, one is asking that $\mathcal{A}$ be a limit cycle), but in this presentation, we restrict ourselves to norms. (The literature usually deals with more general cases. For instance, results on internal stability are often given for $|w(t)|_{\mathcal{A}}$. This generality allows considering issues such as full-state observer design, in which the relevant concepts concern stability with respect to the "diagonal" set $\mathcal{A} = \{(x, x)\}$ where the states of the plant and observer coincide.) Specifically, let us consider i/o systems

$$u(\cdot) \;\to \boxed{\;x(\cdot)\;} \to\; y(\cdot)$$

and various choices of v and w. Roughly, input to state stability arises if $v = u$ and $w = x$, detectability (input and output to state stability) if $v = (u, y)$ and $w = x$, input to output stability when $v = u$ and $w = y$, and output to input stability ("minimum-phase system") when $v = y$ and $w = u$.

The main results to be discussed include characterizations of these properties in terms of dissipation (i.e., Lyapunov-like) inequalities, and superposition theorems (asymptotic characterizations), as well making making systems to be ISS under appropriate feedback laws. The main focus will be on theoretical results, but some applications will be briefly discussed as well. Many open problems will be posed.

Some References

The papers listed below are written by the speaker and several co-authors, as indicated. They may all be obtained from the speaker's web site:

`http://www.math.rutgers.edu/~sontag`

Many other relevant papers can also be obtained from that web site.

The first of the papers provides an outline of some of the topics that will be treated in the lectures, although developments during the last 3 years are not included there. The paper also has as a fairly large list of references, including many to applications papers, again as of 3 years ago. (An updated list of applications papers will be posted to the speaker's web site before the Course.)

1. "The ISS philosophy as a unifying framework for stability-like behavior," in *Nonlinear Control in the Year 2000* (Volume 2) (Lecture Notes in Control and Information Sciences, A. Isidori, F. Lamnabhi-Lagarrigue, and W. Respondek, eds.), Springer-Verlag, Berlin, 2000, pp. 443-468.

2. "Smooth stabilization implies coprime factorization," *IEEE Trans. Automatic Control*, **34**(1989): 435-443.

3. (with Y. Wang) "On characterizations of the input-to-state stability property," *Systems and Control Letters* **24**(1995): 351-359.

4. (with Y. Lin and Y. Wang) "A smooth converse Lyapunov theorem for robust stability," *SIAM J. Control and Optimization* **34**(1996): 124-160.

5. (with A. Teel) "Changing supply functions in input/state stable systems," *IEEE Trans. Autom. Control* **40**(1995): 1476-1478.

6. (with Y. Wang) "New characterizations of input to state stability," *IEEE Trans. Autom. Control* **41**(1996): 1283-1294.

7. (with Y. Wang) "Output-to-state stability and detectability of nonlinear systems," *Systems and Control Letters* **29**(1997): 279-290.

8. "Comments on integral variants of ISS," *Systems and Control Letters* **34**(1998): 93-100.

9. (with D. Angeli) "Forward completeness, unboundedness observability, and their Lyapunov characterizations," *Systems and Control Letters* **38**(1999): 209-217.

10. (with D. Angeli and Y. Wang) "A characterization of integral input to state stability," *IEEE Trans. Autom. Control* **45**(2000): 1082–1097.

11. (with Y. Wang) "Notions of input to output stability," *Systems and Control Letters* **38**(1999): 235-248.

12. "Clocks and insensitivity to small measurement errors," *Control, Optimisation and Calculus of Variations* **4**(1999): 537-557.

13. (with Y. Wang) "Lyapunov characterizations of input to output stability," *SIAM J. Control and Opt.* **39**(2001): 226-249.

14. (with L. Grune and F.R. Wirth) "Asymptotic stability equals exponential stability, and ISS equals finite energy gain - if you twist your eyes," *Systems and Control Letters* **38**(1999): 127-134.

15. (with D. Angeli and Y. Wang) "Further equivalences and semiglobal versions of integral input to state stability," *Dynamics and Control* **10**(2000): 127-149.

16. (with M. Krichman and Y. Wang) "Input-output-to-state stability," *SIAM J. Control and Optimization* **39**(2001): 1874-1928.

17. (with D. Liberzon and A.S. Morse) "Output-input stability and minimum-phase nonlinear systems," *IEEE Trans. Autom. Control* **47**(2002): 422–436.

18. (with M. Krichman) "Characterizations of detectability notions in terms of discontinuous dissipation functions," *Int. J. Control* **75**(2002): 882 - 900.

19. (with B. Ingalls and Y. Wang) "An infinite-time relaxation theorem for differential inclusions," *Proc. Amer. Math. Soc.* **131**(2003): 487–499. (A Banach-space generalization of this result can be found in: "A relaxation theorem for differential inclusions with applications to stability properties," in *Mathematical Theory of Networks and Systems* (D. Gilliam and J. Rosenthal, eds.), 12 pages, August 2002, Electronic Proceedings of MTNS-2002 Symposium held at the University of Notre Dame.)

20. (with M. Arcak and D. Angeli) "A unifying integral ISS framework for stability of nonlinear cascades," *SIAM J. Control and Opt.* **40**(2002): 1888–1904.

21. (with D. Liberzon and Y. Wang) "Universal construction of feedback laws achieving ISS and integral-ISS disturbance attenuation," *Systems and Control Letters* **46**(2002): 111–127.

22. (with D. Angeli and Y. Wang) "Input-to-state stability with respect to inputs and their derivatives," *Intern. J. Robust and Nonlinear Control*, to appear.

23. (with D. Angeli, B. Ingalls and Y. Wang) "Uniform global asymptotic stability of differential inclusions," submitted.

24. (with D. Angeli, B. Ingalls and Y. Wang) "Separation principles for input-output and integral-input to state stability," submitted.

25. (with M. Malisoff, and L. Rifford) "Asymptotic controllability implies input to state stabilization," submitted.