Local Holomorphic Dynamics

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A *discrete local holomorphic dynamical system* is a locally defined holomorphic self-map $f: U \to M$ with a fixed point $p_0 \in U$, where $U \subseteq M$ is an open subset of a complex manifold M.

We shall study the structure of the *stable set* K_f of f, defined as the set of points $p \in U$ whose entire *orbit* $O^+(p) = \{f^k(p) \mid k \in \mathbb{N}\}$ is well-defined, where f^k is the k-th iterate of f (in other words, $p \in K_f$ if and only if $f^k(p) \in U$ for all $k \in \mathbb{N}$). Clearly $p_0 \in K_f$, and K_f is *completely f-invariant*, that is $f^{-1}(K_f) = f(K_f) = K_f$; we are interested in describing both the topological structure of K_f and the dynamical nature of the (global) dynamical system $(K_f, f|_{K_f})$. Similar questions can be asked for less regular (e.g., continuous) local dynamical systems; but we shall see that the answers strongly depend on the holomorphicity of the system.

One important way to study a local holomorphic dynamical system consists in replacing it by an equivalent but simpler system. We shall consider three equivalence relations (topological conjugacy, holomorphic conjugacy, and formal conjugacy), and discuss normal forms and invariants in all these cases.

We shall start discussing the one-dimensional theory, which is fairly complete (even though there are still some open problems), and then we shall present what is known in the multidimensional case, an exciting mixture of deep theorems and very natural questions still unanswered.

References

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