FONDAZIONE C.I.M.E. -- Roberto CONTI
INTERNATIONAL MATHEMATICAL SUMMER CENTER
2010 COURSES

Control of Partial Differential Equations
Cetraro (CS), July 19 -23

This CIME corse addresses several aspects of the control, stabilization and numerical approximation of partial differential equations. It will consist of five series of lectures and a few seminars that will describe the main ideas and techniques developed over the last two decades.

Course Directors

P. Cannarsa (Univ. Roma -Tor Vergata, Italy)
J.-M. Coron (Univ. Pierre et Marie Curie, Paris, France)

Lectures and Abstracts

Fatiha Alabau-Boussouira (Univ. P. Verlaine, Metz, France), *Stabilization of evolution equations.*
Abstract: The series of lectures on stabilization will introduce:

- general methods for stabilization:
- Lasalle's principle and Lyapunov method;
- integral inequalities methods;
- Riccati's method for contruction of feedback laws.

Then, we will develop:

- energy type methods using multipliers for linear and nonlinear stabilization with applications to PDEs with explicit frictional and memory feedbacks;
- indirect stabilization of coupled systems with applications.

If time permits, we will present rapid stabilization of PDEs.

Roger Brockett (Harvard Univ., USA) *Control and optimization related to the Liouville equation.*
Abstract: It has been argued for some time now that the Liouville equation is the proper model for a variety of finite dimensional control problems. This point of view is accepted in the field of quantum control and is gaining acceptance in some more traditional areas as well. This

series of talks intends to focus on the formulation of control problems involving the Liouville equation and their relationship with both stochastic control and quantum control. Results on the controllability and optimization, with and without side constraints, will be discussed. Finally, there will be a discussion of an optimal stabilization question which has motivated and guided some of this work over the last 15 years.

Oleg Emanouilov (Colorado State Univ., USA), *Carleman estimates: applications to controllability and inverse problems.*
Abstract: We give the overview of a general theory of Carleman estimates for partial differential equations. Then we consider applications of these estimates to controllability of PDE such as the Navier-Stokes system, the heat equation, the Burgers equation... Also, we describe a recent result on unique determination of coefficients for elliptic equations obtained by a technique based on Carleman estimates.

Olivier Glass (Univ. Paris-Dauphine, Paris, France), *Control problems in fluid mechanics.*
Abstract: Among the different topics studied in the control theory of Partial Differential Equations, an important one consists in models of fluid mechanics. We will describe several methods used to study both controllability and asymptotic stabilization for some of these systems. The equations that we will consider are of several different types: they include Euler equation (both incompressible in two or three dimensions or compressible in one dimension), viscous Burgers equation, and Korteweg-de Vries equation. We will also consider problems of vanishing viscosity and zero-dispersion in the context of control problems. Several different techniques are in order, such as the return method, Carleman estimates, and the theory of conservation laws.

Enrique Zuazua (Basque Center for Applied Mathematics, Bilbao, Spain), *Control, stabilization and numerics for Partial Differential Equations.*
Abstract: In this series of lectures we shall discuss several topics related with the control, stabilization and numerical approximation of Partial Differential Equations (PDE). We shall mainly focus on the two prototypical examples: wave and heat equations. As it is classical in control theory, we shall address the control problem through its dual, the so called observability problem. It concerns the possibility of measuring or observing by suitable sensors the whole dynamics of the system through partial measurements made on the region in which the controllers are located. This problem is also relevant in other contexts like inverse problems and identification issues. The problem of stabilization, that of producing the exponential decay of solutions by means of suitable feedback damping terms can also be related to the previous ones. In particular, for a broad class of systems, the observability property suffices to build feedback stabilizers.

In these lectures we shall also present our recent work on numerical approximation problems. We shall show that, due to high frequency spurious numerical solutions, stable algorithms for solving initial-boundary value problems do not necessarily yield convergent algorithms for computing the controls. These phenomena arise for wave equations, due to the existence of undamped high frequency numerical spurious solutions. Such solutions do not exist for heat equations in one space dimension since the intrinsic smoothing and damping effect of parabolic models suffices to rule these spurious solutions out. But the same is not true for the

multi-dimensional heat equation. Several examples will be discussed showing that a complete theory should take into account issues related with the high frequency behavior of the numerical schemes, the properties of the underlying PDE, the control problem under discussion and the geometry of the domains under consideration. We shall then analyze in some detail the wave equation and develop a number of remedies to avoid those high frequency oscillations and restablish the uniform (with respect to the mesh-size) control, observation and stabilization properties. In particular, we shall describe a two-grid algorithm introduced by R. Glowinski which works in a more systematic way. We shall also interpret these results in microlocal terms, in connection with the dynamical properties of the bicharacteristic flow. Connections will also be done with the theory of waves in heterogenous media. We shall later discuss the heat equation. It is by now well known that control properties of the wave equation can be immediately transformed into control results for the heat equation. This can be done using Kannai's transforms or Russell spectral principle. We shall therefore, to start, discuss the consequences of our previous analysis on the wave equation for the heat one. We shall explain the main obstacles that arise when looking for the appararently most natural (and false!) statement saying that numerical approximation schemes for the control of the heat equation should converge without geometric restrictions on the support of the control. The continuous analog is by now well known and has been proved using Carleman inequalities. We shall discuss the main dificulties arising when deriving such inequalities for discrete operators.

This series of lectures will conclude with a list of challenging open problems and subjects for future research.