TITLE: TOPICS IN MATHEMATICAL FLUID-MECHANICS.

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Prof. P.Constantin: Fluids and Particles.

Short description: We are going to consider mathematical descriptions of complex fluids, in which particles and fluids interact. The particles are carried by the fluid, thermally agitated and interact among themselves. The particles add stress to the fluid system and drive the fluids. The thermodynamic equilibrium of the particles is described by Onsager's equation, a nonlocal nonlinear equation for the probability distribution of particles on a configuration space. The kinetic description of interacting particles is done via certain nonlinear Fokker-Planck equations, the Smoluchowski equations. The complex fluids system is formed with Navier-Stokes equations coupled with nonlinear Fokker-Planck equations. Thermodynamical considerations allow to derive the effect that the fluid has on the particles (the macro-micro effect). The derivation requires the assumption that the fluid is smooth on the scale of the particles. By contrast, the effect that the particles have on the fluid (the micro-macro effect) is a highly nontrivial and deep question, that will be addressed using energetic considerations. Hybrid systems will be also described, in which stochastic particle equations are coupled to fluid equations. These are models for complex fluids, related to the determinsitic system of Navier-Stokes equations coupled to Nonlinear Fokker Planck equations.
The lessons will be concerned with mathematical questions of existence, stability, asymptotic behavior and traveling waves. They will be divided in three major components:
1. Particles in equilibrium. 2. Dynamics of particles and fluids 3. Stochastic aspects.

Prof. G.P. Galdi: Topics in the Mathematical Theory of Fluid-Solid Interaction.

Even though problems of fluid-solid interaction are more or less ubiquitous in many branches of applied science ranging from a small to a large scale a systematic mathematical treatment of some of their relevant and basic aspects has begun only a few years ago. This late start is probably due to the intrinsic, severe difficulty of the relevant equations. In fact, the presence of the solid (rigid or elastic) affects the flow of the liquid, and this, in turn, affects the motion of the solid, so that the problem of determining the flow characteristics is highly linked, typically, through a non-local coupling. It is just this latter feature that makes any fundamental mathematical problem related to fluid-solid interaction a particularly challenging one. The role of mathematics in the investigation of these problems is twofold and is directed toward the accomplishment of the following objectives. The first one, of a more theoretical nature, is the validation of the models proposed by engineers, and consists in securing conditions under which the governing equations possess the fundamental requirements of well-posedness, such as existence and uniqueness of corresponding solutions and their continuous dependence upon the data. The second one, of a more applied character, is to prove that these models give a satisfactory interpretation of the observed phenomena. In general, both tasks present serious difficulties in that they require the study of several different, and frequently combined, topics that include, among others, Navier-Stokes equations, non-Newtonian fluid models, nonlinear elasticity, and multi-phase flow. It must be added that some of these topics are still at the beginning of a systematic mathematical research, whereas some others are in a continuous growth. The objective of the this short course

is to provide a mathematical analysis of some topics of fluid-solid interaction. Specifically, it aims at covering the following subjects.
1. Motion of a rigid body in Newtonian and viscoelastic liquid under pre- scribed forces and torques. General Theory.
2. Analysis of the well-posedness of the basic initial and boundary-value problem associated to the motion of a rigid body in a Newtonian fluid.
3. Motion of a Newtonian and viscoelastic fluid in an elastic cavity. Analysis of the corresponding boundary-value problem.
4. Steady flow of a Newtonian fluid past an elastic body. 5. Self-propelled motion of a deformable body in a Newtonian liquid.

Prof. M. Ruzicka: Analysis of generalized Newtonian fluids.

Many incompressible fluids that can not adequately described by a a linear constitutive law (Newtonian fluids) fall into the class of generalized Newtonian fluids, where one works with a power law like constitutive relation. In the last years there was a lot of intensive and fruitful work on such fluids. This includes substantial improvements of the existence theory for weak and strong solutions, as well as to the numerical analysis of appropriate approximations, including space and time discretizations.
In the talks we plan to present recent results on the existence of weak solutions to the system describing the motion of incompressible generalized Newtonian fluids using the Lipschitz truncation method. Moreover we will show the existence of local in time strong solutions, which is the basis for optimal error estimates. Finally we will present an error analysis for space and time discretizations of the equations, which provides optimal error estimates without dependence on the growth parameter of the constitutive relation.

Prof. G. Seregin: Local Regularity Theory for the Navier-Stokes Equations.

The aim of the course is to explain how one can develop the local regularity theory for the three-dimensional nonstationary Navier-Stokes equations. Even in the absence of nonlinear term, such a theory differs essentially from the usual local regularity theory for the classical PDE's of elliptic and parabolic types. We shall discuss the so-called Caffarelli-Kohn-Nirenberg type theory for suitable weak solutions based on the Navier-Stokes scaling and scaled-invariant functionals. Among more recent topics, it will be shown how one can construct ancient solutions to the Navier-Stokes equations with different kinds of blow up at possible singular points. We shall discuss some problems of unique continuation and Liouville's theorems for those ancient solutions.

Prof. Edriss S. Titi: Mathematical Analysis of Certain Geophysical Models and Sub-grid Scale Models of Turbulence.

In this series of lectures I will be covering two main topics. The first part will be concerning certain geophysical flows. The second part of my lectures will address analytical sub-subgrid scale models of turbulence.

1. **Geophysical Flows:** The basic problem faced in geophysical fluid dynamics is that a mathematical description based only on fundamental physical principles, which are called the “Primitive Equations”, is often prohibitively expensive computationally, and hard to study analytically. In this talk I will survey the main obstacles in proving the global regularity for the three dimensional Navier–Stokes equations and their geophysical counterparts. However, taking advantage of certain geophysical balances and situations, such as geostrophic balance and the shallowness of the ocean and atmosphere, geophysicists derive more simplified and manageable models which are easier to study analytically. In particular, I will present the global well-posedness for the three dimensional Benard convection problem in porous media, and the global regularity for a three-dimensional viscous planetary geostrophic models. Furthermore, these systems will be shown to have finite dimensional global attractors. Based on the tools developed for attacking these problems I will also prove the global regularity for the three-dimensional Primitive equations of large scale oceanic and atmospheric dynamics.

2. **Analytical Sub-grid Scale Models of Turbulence:** In recent years many analytical sub-grid scale models of turbulence were introduced based on the Navier–Stokes-alpha model (also known as a viscous Camassa–Holm equations or the Lagrangian Averaged Navier–Stokes-alpha (LANS-alpha)). Some of these are the Leray-alpha, the modified Leray-alpha, the simplified Bardina-alpha and the Clark-alpha models. In this part I will show the global well-posedness of these models and provide estimates for the dimension of their global attractors, and relate these estimates to the relevant physical parameters. Furthermore, I will show that up to certain wave number in the inertial range the energy power spectra of these models obey the Kolmogorov -5/3 power law, however, for the rest the inertial range the energy spectra are much steeper.

In addition, I will show that by using these alpha models as closure models to the Reynolds averaged equations of the Navier–Stokes one gets very good agreement with empirical and numerical data of turbulent flows in infinite pipes and channels.

It is also observed that, unlike the three-dimensional Euler equations and other inviscid alpha models, the inviscid simplified Bardina model has global regular solutions for all initial data. Inspired by this observation we introduce new inviscid regularizing schemes for the three-dimensional Euler and Navier–Stokes equations, which does not require, in the Navier–Stokes case, any additional boundary conditions. This same kind of inviscid regularization is also used to regularize the Surface Quasi-Geostrophic model.

Finally, and based on the alpha regularization we will present new approximation of vortex sheets dynamics.
