Resonance problems for some non-autonomous ordinary differential equations

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Recent years have seen a lot of activity in the study of quasilinear non-autonomous ordinary differential equations of the form

$$(\phi(y'))' = f(t, y, y') \tag{1}$$

where $\phi : A \subset \mathbb{R} \to B \subset \mathbb{R}$ is an increasing homeomorphism such that $\phi(0) = 0$ between the open sets A and B. The situation generalizes the classical case where $A = B = \mathbb{R}$ and ϕ is the identity, and the well-studied case of the p-Laplacian ($p > 1$) where $\phi(s) = |s|^{p-2}s$.

In this last case, the Fredholm alternative for the solvability of

$$(|y'|^{p-2}y')' - \lambda|y|^{p-2}y = h(t) \tag{2}$$

with classical Dirichlet, Neumann or periodic boundary conditions on $[0, T]$

$$x(0) = 0 = x(T), \quad x'(0) = 0 = x'(T), \quad x(0) - x(T) = 0 = x'(0) = x'(T) \tag{3}$$

continues to pose open problems, despite of recent partial results which will be described.

Contemporary researches concern less standard situations where $\phi : (-a, a) \to \mathbb{R}$ (singular homeomorphism) and $\phi : \mathbb{R} \to (-a, a)$ (bounded homeomorphism). A model for the first case, namely $\phi(s) = \frac{s}{\sqrt{1-s^2}}$, corresponds to acceleration in special relativity. A model for the second situation, namely $\phi(s) = \frac{s}{\sqrt{1+s^2}}$, corresponds to problem with curvature satisfying various conditions. In those case, both topological and variational methods, and sometimes combination of them give new complementary existence and multiplicity results. We will describe some of them.

Some attention will be given to the generalized forced pendulum equation

$$(\phi(y'))' + A \sin y = h(t) \tag{4}$$

when ϕ is singular or bounded.

The case of differential systems, with or without variational structure, will be considered as well.