Pluripotential theory and bifurcations in holomorphic dynamics

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These lectures are devoted to the study of bifurcations within holomorphic families of rational maps or polynomials by mean of ergodic and potential theoretic tools. The first lecture will give a general overview of the subject. The remaining five lectures will be organized as follows.

1. Rational functions as ergodic dynamical systems
We introduce the Green measure μ_f of a rational map f and study the properties of its Lyapunov exponent $L(f)$.

2. Holomorphic families
We introduce the general setting and present two important objects which will then play a central role. The first one is a class of hypersurfaces $(Per_n(w))$ in the parameter space of a holomorphic family which are defined by the existence, for the corresponding rational map, of a cycle with a given period n and a given multiplier w. The second object is the connectedness locus in polynomial families.

3. The bifurcation current
We introduce the bifurcation current T_{bif}. It is a positive $(1,1)$ closed current on the parameter space of a holomorphic family. We show T_{bif} exactly detects the bifurcations in the classical sense of the Mañé-Sad-Sullivan theory and admits the Lyapunov exponent function as a global potential : $T_{\text{bif}} = dd^c L$.

4. Equidistribution towards the bifurcation current
We will see that certain dynamically defined hypersurfaces (in particular the $Per_n(w)$) equidistribute the bifurcation current T_{bif}. This will allow us to discover and precisely describe some laminated structures in bifurcation loci.

5. The bifurcation measure
We investigate the self-intersection of the bifurcation current. In particular, we introduce a measure μ_{bif} , called bifurcation measure, which detects stronger bifurcations than the current. This measure is the Monge-Ampère mass of the Lyapunov function L. We give a precise description of the support of μ_{bif} in terms of Misiurewicz or Shishikura parameters.