Interaction of a Navier-Stokes Liquid with a Rigid Body

Giovanni P. Galdi, Pittsburgh USA

In addition to their greatest significance from a strict mathematical viewpoint, the Navier-Stokes equations are a fundamental tool in understanding a number of phenomena that are of major relevance in the applied sciences. As a matter of fact, the primary reason why these equations were originally proposed by C. Navier in 1822 was to give a quantitative explanation of a phenomenon that could not be covered by the inviscid theory of D'Alembert and Euler. The challenge consisted in the evaluation of the resistance of water against ships and pipe walls and, more generally, the study of the interaction of a liquid with a rigid body.

Besides its important role in physical and engineering disciplines, this type of study generates a number of mathematical questions, which are, at the same time, intriguing and challenging.

Objective of these lectures is to give a fairly complete investigation of two broad classes of problems regarding the interaction of a viscous liquid with a rigid body, and furnish a rigorous mathematical interpretation of certain basic phenomena observed in lab experiments.

The one class consists of the coupled motion of a rigid body with a cavity entirely filled with a Navier-Stokes liquid. Here, the characteristic feature is that the liquid generates a substantial stabilizing effect on the motion of the body that, surprisingly enough, can even bring the body to rest. The other class, instead, concerns the classical phenomena of self-oscillation observed in a viscous flow past a rigid obstacle that, mathematically, falls in the category of time-periodic bifurcation from a steady-state.

As we shall show, in spite of their substantial difference, both classes of (nonlinear) problems share a common mathematical property, namely, the spectrum of the relevant linearized operators has a non-empty intersection with the imaginary axis, regardless of the "magnitude" of the appropriate dimensionless numbers. It is just this aspect that makes the analysis interesting and, at times, rather difficult.

These lectures will also point out several open questions that may be of great appeal to the interested mathematician.

References.

1. Galdi, G.P., On bifurcating time-periodic flow of a Navier–Stokes liquid past a cylinder. *Arch. Ration. Mech. Anal.* **222** (2016), no. 1, 285-315

2. Disser, K., Galdi, G.P., Mazzone, G., and Zunino, P., Inertial motions of a rigid body with a cavity filled with a viscous liquid. *Arch. Ration. Mech. Anal.* **221** (2016), no. 1, 487-526.

3. Galdi, G.P., and Mazzone, G., On the motion of a pendulum with a cavity entirely filled with a viscous liquid. Recent progress in the theory of the

Euler and Navier-Stokes equations, 37-56, *London Math. Soc. Lecture Note Ser.*, **430**, Cambridge Univ. Press, Cambridge, 2016

4. Galdi, G.P. and Neustupa, J., Steady-State Navier–Stokes Flow Around a Moving Body, *Handbook of Mathematical Analysis in Mechanics of Viscous Fluids*, Springer-Verlag, 1-77 (2016)

5. Galdi, G.P., Stability of permanent rotations and long-time behavior of inertial motions of a rigid body with an interior liquid-filled cavity, Particles in Flow, Birkhäuser-Verlag (2017)

Analysis of Incompressible Viscous Fluid Flow: an Approach by Maximal L^p-Regularity

Matthias Hieber

TU Darmstadt, Germany

In this series of lectures, we consider various models from fluid dynamics ranging from the equations of Navier-Stokes over the Ericksen-Leslie system for nematic liquid crystal flow and two-phase free boundary value problems as well as fluid-structure interaction problems for complex fluids to the primitive equations of geophysical flows.

The approach presented relies on so the called *maximal L^p-regularity property* of the linearized equation and on modern theory of quasilinear parabolic equations. It is hence natural to start this lecture series with an introduction to the characterization of the maximal L^p-regularity property of the solution of the linearized equation in terms of the $\mathcal{R}$-boundedness of the resolvent of the associated generator. To this end, we introduce in addition the concepts of operator-valued Fourier multipliers and *operator-valued H^∞-calculus*.

We then show that these methods yield local, strong well-posedness results for *abstract quasilinear parabolic systems*, as well as global existence results for strong solutions for small data under suitable assumptions on the spectrum of the linearization and the nonlinearities.

The above methods and techniques are then applied to various classes of models arising in *incompressible, viscous fluid flow* and, in particular, to the ones described already above. More specificially, we analyze the *Ericksen-Leslie model describing nematic liquid crystal flow* and following this approach we obtain a rather complete picture of the dynamics of this system.

Next, we will discuss *two-phase free boundary value problems*, subject to surface tension and gravitational forces, for various classes of Newtonian and non-Newtonian fluids and show strong well-posedness results for this system within the L^p-setting. Results of similar type will in addition be discussed for *fluid-rigid body type interaction* problems.

Finally, we apply our approach also to the *primitive equations of geophysical flows.* In this case, due to a suitable a-priori estimate, we even obtain the existence of a unique, strong solution for arbitrary large data belonging to certain fractional power spaces of the hydrostatic Stokes operator.

Method of the Besov space and its applications to the strong solutions of the Navier-Stokes equations

Hideo KOZONO

Department of Mathematics, Waseda University, Tokyo, Japan
kozono@waseda.jp

Abstract

Since the pioneer work of Kato's L^p-strong solution, a number of efforts have been made to enlarge the space of the initial data which enables us to obtain the local existence of strong solutions to the Navier-Stokes equations. For instance, $L^n(\mathbb{R}^n)$, $L^{n,\infty}(\mathbb{R}^n)$, $M^n(\mathbb{R}^n)$, $\dot{B}_{p,\infty}^{-1+n/p}(\mathbb{R}^n)$ and $\dot{F}_{\infty,2}^{-1}(\mathbb{R}^n)$ are monotonically increasing function spaces of initial data in which the local well-posedness of the Navier-Stokes equations has been clarified. In this series of lectures, we bring a focus onto the Besov spaces and discuss local and global well-posedness in the scaling invariant cases. In particular, we deal with the suitable class of external forces. Our final goal is to find the largest homogeneous Besov space where the well-posedness is established for both initial data and external forces. To this end, we make fully use of the maximal Lorentz regularity theorem in Besov spaces.

Contents of the course

(i) Strong L^p-solutions

(ii) Short introduction to the Besov space

(iii) $L^p - L^q$-estimates for the semigroup and bilinear estimates in Besov spaces

(iv) Maximal Lorentz regularity theorem of the Stokes equations in Besov spaces

(v) Well-poseness of the Navier-Stokes equations in Besov spaces

Lecture on some free boundary problem for the Navier Stokes equations

By Yoshihiro SHIBATA*

In this lecture, we study some free bounary value problems for the Navier-Stokes equations. The local wellposedness, the global wellposedness, and asymptotics of solutions as time goes to infinity are treated in the L_p in time and L_q in space framework. The tool in proving the local well-posedness is the maximal L_p-L_q regularity for the Stokes equations with non-homogeneous free boundary conditions. To obtain the maximal L_p-L_q regularity, we use the $\mathcal{R}$ bounded solution operators of the generalized resolvent problem for the Stokes equations with non-homogeneous free boundary conditions and the Weis operator valued Fourier multiplier. Our method is useful to prove the maximal L_p-L_q regularity theory of the initial boundary value problem not only for linear systems of parabolic equations but also hyperbolic-parabolic coupled systems like Stokes equations for the compressible viscous fluid flows with non-homogeneous boundary conditions in a general unbounded domains with some uniformity of the shape of boundary, so called uniform C^m domains.

To prove the global well-posedness for the strong solutions, the key issue is some decay properties of Stokes semigroup, which are derived by the spectral analysis of the Stokes operator in the bulk space and the Laplace-Beltrami operator on the boundary. In this lecture, we treat the following two cases: (1) a bounded domain with surface tension and (2) an exterior domain without surface tension. In particular, in treating the exterior domain case, it is essential to choose different exponents p and q. Because, in the unbounded domain case, we can obtain only polynomially decay in suitable L_q norm in space, and so to guarantee the integrability of L_q norm of solutions in time, it is necessary to choose an exponent p suitably large.

The lecture consists of four parts. In the first lecture, the problem is introduced and some transformations are explained. Because, free boundary problems treated in this lecture are described by Navier-Stokes equations in time dependent domains , which are unknown. Thus, it is necessary to transform such unknown domains to some known fixed domains. In the second lecture, the maximal L_p-L_q regularity for the Stokes equations with free boundary conditions is explained. In the third lecture and fourth lecture, we prove the global well-posedness in the case where the reference domain is a bounded one and the surface tension is taken into account. In the last lecture, the global well-posedness is exlained in the case where the reference domain is an exterior one and the surface tension is not taken into account.

References

[1] J. Pruess and G. Simonett, Moving Interfaces and Quasilinear Parabolic Evolution Equations, Birkhauser Monographs in Mathematics, 2016, ISBN: 978-3-319-27698-4

[2] Y. Shibata, On the $\mathcal{R}$-boundedness of solution operators for the Stokes equations with free boundary condition, Diff. Int. Eqns **27**(3-4) (2014), 313–368.

[3] Y. Shibata, *On some free boundary problem of the Navier-Stokes equations in the maximal L_p-L_q regularity class*, J. Differential Equations **258** (2015), 4127–4155.

[4] Y. Shibata, *Global well-posedness of unsteady motion of viscous incompressible capillary liquid bounded by a free surface*, to appear in EECT.

[5] Y. Shibata, *Local wellposedness for the free boundary problem of the Navier-Stokes equations in an exterior domain* , Submitted.

[6] Y. Shibata, *On L_p-L_q decay estimate for Stokes equations with free boudary condition in an exterior domain*, Submitted.

[8] Y. Shibata, *Global wellposedness of a free boundary problem for the Navier-Stokes equations in an exterior domain*, Submitted.

*Department of Mathematics and Research Institute of Science and Engineering, Waseda University

[9] V. A. Solonnikov, *On the transient motion of an isolated volume of viscous incompressible fluid*, Math. USSR Izvestiya **31** (1988) (2), 381–405.

[10] V. Solonnikov, *Unsteady motion of a finite mass of fluid, bounded by a free surface*, J. Soviet Math., **40** (1988), 672–685.

[11] L. Weis, *Operator-valued Fourier multiplier theorems and maximal L_p-regularity*, Math. Ann. **319** (2001), 735–758.

Partial regularity for the 3D Navier Stokes equations and applications

James Robinson, Warwick UK

In these lectures I will cover some conditional "epsilon regularity" results and the partial regularity results they imply, which limit the size of the set of possible singularities. I will discuss what room there might be for improvement in these results, and investigate some of their consequences.

I will start with the result that the box-counting dimension of the possible singular times of a weak solution is no larger than $1/2$; such results are relatively simple and date back to Scheffer (1977), although the box-counting variant I will prove here is due to Robinson & Sadowski (2007). I will show that no better bound is possible using only the simple ingredients that go into the proof.

The main topic will be a full proof of a conditional regularity result due to Caffarelli, Kohn, & Nirenberg (1982), broadly following their original method: there exists an absolute constant $\varepsilon_1 > 0$ such that

$$\frac{1}{r^2}\int_{Q_r} |u|^3 + |p|^{3/2} < \varepsilon_1$$

implies that $u \in L^\infty(Q_{r/2})$, where $Q_r = B_r(x) \times (t - r^2, t)$. A consequence of this is that box-counting dimension of the set of space-time singularities can be no larger than $5/3$.

I will briefly discuss a second conditional regularity result under the condition

$$\limsup_{r\to 0} \frac{1}{r}\int_{Q_r} |\nabla u|^2 < \varepsilon_2;$$

this can be used to show that the Hausdorff dimension of the set of space-time singularities is no larger than 1. An involved construction due to Scheffer (1985, 1987) shows that no better bound is possible without using more information about solutions of the Navier-Stokes equations than is used in the proof of CKN.

Finally I will show how the bound on the box-counting dimension of the set of space-time singularities can be used to show that particle trajectories (i.e. solutions of $\dot{X} = u(X, t)$) are unique for almost every choice of initial condition $X(0)$ whenever u is a weak solution (Robinson & Sadowski, 2009). This means that one can use a Lagrangian description of the fluid flow even when the solution is not known to be regular.

While much of the material here is classical, the presentation and content will draw heavily on joint work with Wojciech Ozanski (Warwick), Jose Rodrigo (Warwick), Witold Sadowski (Bristol), and Nicholas Sharples (Middlesex).