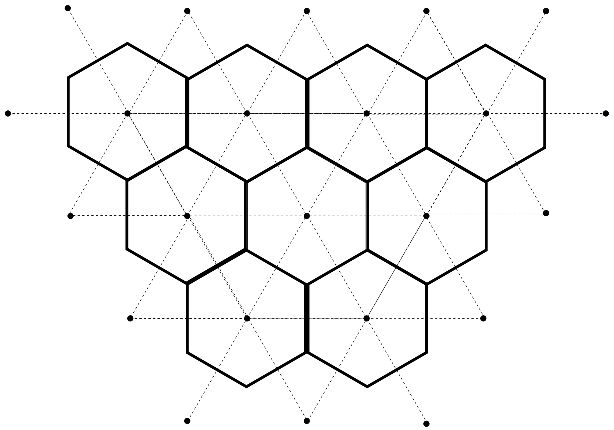
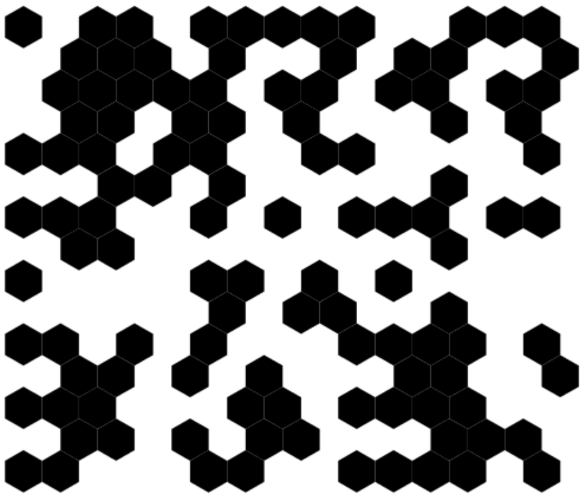
Trivial, critical and near-critical scaling limits in two-dimensional percolation

Joint work with Federico Camia and Ronald Meester

Model

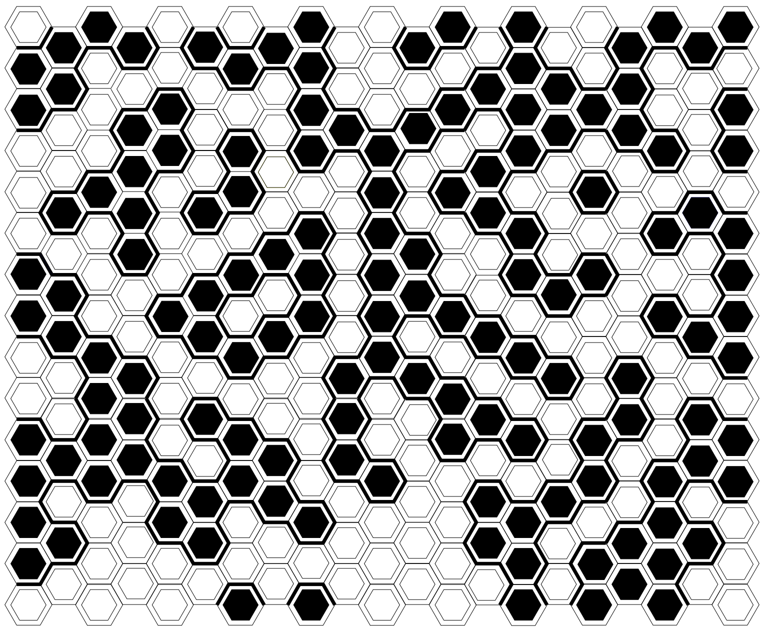


Honeycomb lattice



Hexagons are colored white w.p. p and black w.p. $1 - p$, independently of each other

Boundary curves

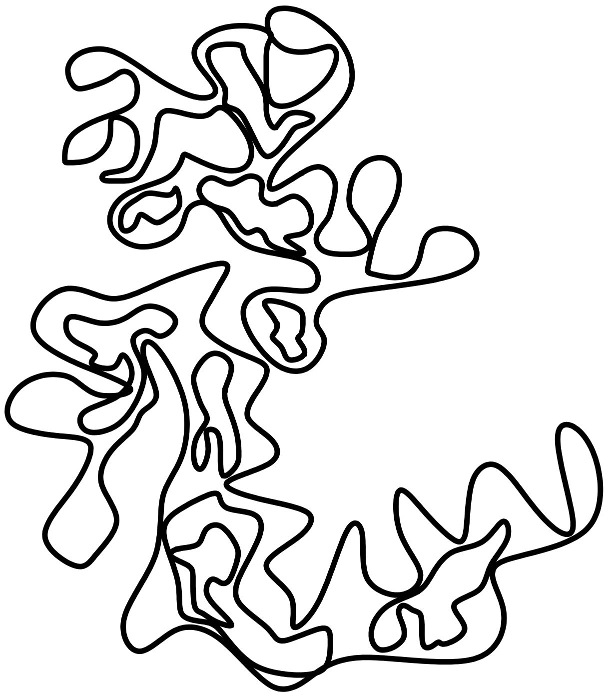


Boundary circuit (loop):= edge-boundary of a monochromatic cluster

Set of boundaries becomes a collection of nested, oriented, simple loops

$\mu_{\delta,p}$= probability measure that "describes" the boundary circuits, induced by percolation with parameter p on the lattice with mesh δ

Critical percolation ($p = 1/2$): continuum scaling limit



The probability distribution $\mu_{\delta,1/2}$ converges weakly, as $\delta \to 0$, to a probability distribution on collections of continuum nonsimple loops in the plane.

Loops are nonsimple but there are no triple points.

Every point in the plane is surrounded by infinitely many loops, with diameters going to both 0 and ∞.

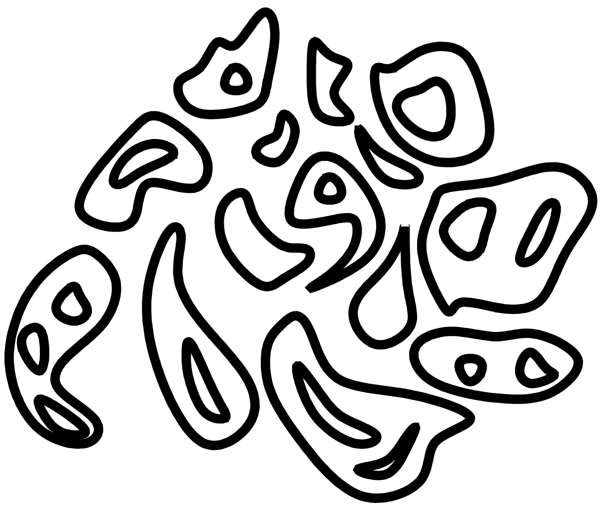
Percolation near $p = 1/2$

Lemma For any sequence $\delta_i \downarrow 0$ and any choice of parameters p_i there exists a probability measure μ which is the weak subsequential limit of $\{\mu_{\delta_i, p_i}\}$

Theorem There are three non-void possibilities for μ:

- Trivial: No loops of diameter larger than zero μ-a.s.
- Critical: μ coincides with the scaling limit of critical percolation.
- Near-critical: μ-a.s. any point is surrounded by infinitely many loops but there exists a largest one.

Geometric properties



At short scale the limit looks critical: infinitely many loops with diameter going to zero.

At large scale it differs from the critical scaling limit: there exists a largest loop around every point, hence it is not scale invariant.

Bonus

For $p \neq 1/2$ there exists a distance $L(p)$, the *correlation length*, such that percolation in a window with length $L(p)$ "looks critical".

Trivial scaling limit is obtained when $\delta_i L(p_i) \to 0$ as $\delta_i \to 0$.

Critical scaling limit is obtained when $\delta_i L(p_i) \to \infty$ as $\delta_i \to 0$.

The near-critical scaling limit will arise when $0 < \delta_i L(p_i) < \infty$ as $\delta_i \to 0$.