Invasion percolation in 2D

Artem Sapozhnikov
(joint work with Michael Damron and Bálint Vágvölgyi)

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Question 3:
What can we say about the distribution of $\mathcal{S}$?

Observations

- Let $p > p_c$. There exists an infinite p-open cluster $\mathcal{C}_p$ with probability 1. If

$$\mathcal{S}_n \cap \mathcal{C}_p \neq \emptyset$$

for some n, then

$$\mathcal{S} \setminus \mathcal{S}_n \subset \mathcal{C}_p;$$

- For any $p > p_c$,

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Theorem

For any $k \geq 1$, there exist C_1 and C_2 such that

$$\mathbb{P}(\hat{D}_k \geq n) \geq C_1 (\log n)^{k-1} \mathbb{P}_{p_c}(diam\ C(0) \geq n)$$

and

$$\mathbb{P}(\hat{D}_k \geq n) \leq C_2 (\log n)^{k-1} \mathbb{P}_{p_c}(diam\ C(0) \geq n).$$

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Theorem

For any $k \geq 1$, there exist C_3 and C_4 such that

$$\mathbb{P}(|\hat{V}_k| \geq n) \geq C_3 (\log n)^{k-1} \mathbb{P}_{p_c}(|C(0)| \geq n)$$

and

$$\mathbb{P}(|\hat{V}_k| \geq n) \leq C_4 (\log n)^{k-1} \mathbb{P}_{p_c}(|C(0)| \geq n).$$

Incipient infinite cluster

- The limit

$$\nu(E) = \lim_{p \downarrow p_c} \mathbb{P}_p(E \mid |C(0)| = \infty)$$

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- Note that $\nu(|C(0)| = \infty) = 1$.

Results

Recall the definition of the invasion percolation cluster $\mathcal{S}$.

Theorem (Jarai, '02)

For any finite subset of edges $\mathcal{K}$ and $x \in \mathbb{Z}^2$,

$$\lim_{|x|\to\infty} \mathbb{P}(x + \mathcal{K} \subset \mathcal{S} \mid x \in \mathcal{S}) = \nu(\mathcal{K} \subset C(0)).$$

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Theorem

The laws of the invasion percolation cluster and the incipient infinite cluster are mutually singular.