Random walk delayed on percolation clusters

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Introduction

We consider a random walk in random environment where the environment is given by a subcritical percolation. The *quenched* process is a continuous random walk in \mathbb{Z}^d subject to a drift and attraction to large clusters of the environment.

Environment

We consider a subcritical i.i.d. site percolation ω on \mathbb{Z}^d : Fix $p < p_c(d)$ and a variable $\omega \in \Omega = \{0, 1\}^{\mathbb{Z}^d}$ with law

 $\mathbb{P} = Ber(p)^{\otimes \mathbb{Z}^d}.$

Notation : C_x will denote the size of the cluster of x ($x \in \mathbb{Z}^d$)

Exponential decay of the cluster size

There exists $\xi(p, d) > 0$ such that

$$-\frac{1}{n}\ln\mathbb{P}(C_0>n)\to\xi>0\qquad(n\to\infty)$$

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The quenched law

Fix an environment ω and a direction $\ell \in S_{d-1}$. We need two parameters :

- $\lambda \geq 0$ for the strength of the drift
- $\beta \geq 0$ for the strength of the attraction by the clusters

We now define the continuous time Markov Chain $(Y_t)_{t\geq 0}$ with law P_{ω} by assigning its skeleton and jump rates.

Skeleton

The skeleton $(X_n)_{n\geq 0}$ is the drifted random walk on \mathbb{Z}^d with transitions

$$ilde{P}(X_{n+1}=x+e|X_n=x)=rac{e^{\lambda\ell\cdot e}}{\sum_{|e'|=1}e^{\lambda\ell\cdot e'}},\quad |e|=1.$$

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The quenched law

- Notice that the skeleton is independent from the environment ω .
- $d(\lambda)$ will denote the drift of $(X_n)_{n\geq 0}$.

Jump rates

The jump rate at site $x \in \mathbb{Z}^d$ is $e^{-\beta C_x}$.

The waiting time at site x are i.i.d. exponential variables with mean $e^{\beta C_x}$.

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Clock Process

Consider a family of i.i.d. exponential variables $(\epsilon_i)_{i \in \mathbb{N}}$ with mean 1 (note Q the law of this family) and define the clock process as :

$$S_n = \sum_{i=0}^{n-1} \epsilon_i e^{\beta C_{X_i}}.$$

We now consider the process

$$Y_t = X_{S^{-1}(t)}, \quad t \ge 0,$$

and define P_{ω} as the law of $(Y_t)_{t\geq 0}$ under $\tilde{P}\otimes Q$.

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Properties P_{ω}

P_{ω} is reversible and admits the reversible measure,

$$\mu_{\omega}(x) = e^{2\lambda\ell \cdot x + \beta C_x}, x \in \mathbb{Z}^d$$

We will also use the annealed law P :

$$P=\mathbb{P}\times P_{\omega}.$$

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The Bouchaud Trap Model

This model can be seen as a BTM on \mathbb{Z}^d (Ben Arous-Cerny)

- The skeleton is still a simple random walk, but now possibly drifted.
- The $(\tau_x)_{x\in\mathbb{Z}^d}$ are not i.i.d. as $\tau_x = e^{\beta C_x}$.

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Popov and Vachkovskaia Model

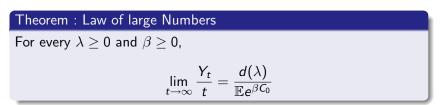
S.Popov et M.Vachkovskaia studied the discrete time walk attracted by clusters with transitions,

$$P(X_{n+1}=x+e|X_n=x)=\frac{e^{\beta C_{x+e}}}{\sum_{|e'|=1}e^{\beta C_{x+e'}}}.$$

 $\mu_{\omega}(x) = e^{\beta C_x}, x \in \mathbb{Z}^d$, is also a reversible measure for this dynamic ...but not for the same reasons.

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Law of large numbers



Main tool of the proof : the environment seen from the particle is an ergodic Markov chain under the annealed law.

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Sub-diffusive case

Theorem : Subdiffusive case : $\beta \geq \xi$ • For every $d \ge 1$ and $\lambda > 0$, $\lim_{t \to \infty} \frac{\ln |Y_t|}{\ln t} = \frac{\xi}{\beta}, \quad P - p.s.$ • If $\lambda = 0$ and d > 2, $\limsup_{t \to \infty} \frac{\ln |Y_t|}{\ln t} = \frac{\xi}{2\beta}, \quad P - p.s.$ • If $\lambda = 0$ and d = 1. $\limsup_{t\to\infty} \frac{\ln|Y_t|}{\ln t} = \frac{1}{2} \left(\frac{\beta}{2\xi} + \frac{1}{2}\right)^{-1}, \quad P - p.s.$

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Quenched Invariance principle

Theorem : Quenched Invariance principle

If
$$\lambda = 0$$
 and $\mathbb{E}(e^{eta \mathcal{C}}) < \infty$, then $\mathbb{P} - p.s.$,

$$(\epsilon^{1/2}Y_{\epsilon^{-1}t})_{t\geq 0} \Rightarrow (B_t)_{t\geq 0}$$

Proof :

- $(Y_t)_{t\geq 0}$ is a martingale under the quenched law.
- Ergodic theorem gives the existence of a limit for the bracket of $(\epsilon^{1/2}Y_{\epsilon^{-1}t})_{t\geq 0}$.

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Conclusions

- Convergence in law as for the BTM.
- Study of the model of Popov and Vachkovkaïa with a drift.
- Study of other Markovian dynamics admitting μ_ω as reversible measure.