

# LA GEOMETRIA IN MESOPOTAMIA



Ms 3050 Schoyen collection  
periodo paleobabilonese

# LA GEOMETRIA IN MESOPOTAMIA

## Tavolette di geometria

Varie centinaia

### **Contenuti**

Misura di aree

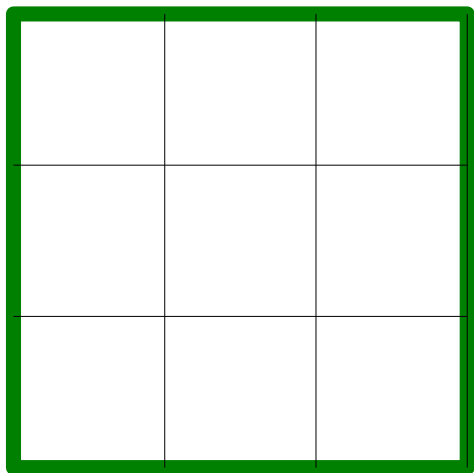
- rettangoli
- triangoli rettangoli
- trapezi e quadrilateri
- cerchi

+ alcuni risultati avanzati

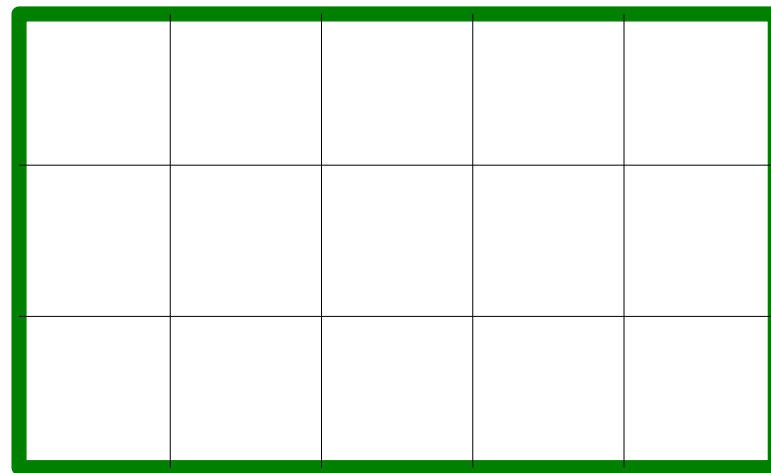
Poche differenze lungo tutto il periodo Sumero-Babilonese:  
3000-1500

Un sapere che rimane simile per quasi due millenni

# QUADRATI e RETTANGOLI



$$A=l \times l$$



$$A=b \times h$$

## Misure di lunghezza

Unità di misura	Nome sumero	Nome accadico	Equivalenza	Quantità
Dito	SU.SI	ubanum		1,6 cm
Cubito	KUS	ammatum	30 dita	circa 50 cm
Canna	GI	qanum	6 cubiti	circa 3 metri
	NINDA		2 GI	circa 6 metri
	US		60 NINDA	circa 360 metri
	DANNA	berum	30 US	circa 11 Km

# Misure di superficie

Unità di misura	Nome sumero	Nome accadico	Equivalenza	Quantità
-----------------	-------------	---------------	-------------	----------

	SAR	musarum	1 NINDA al quadrato	circa 36 mq
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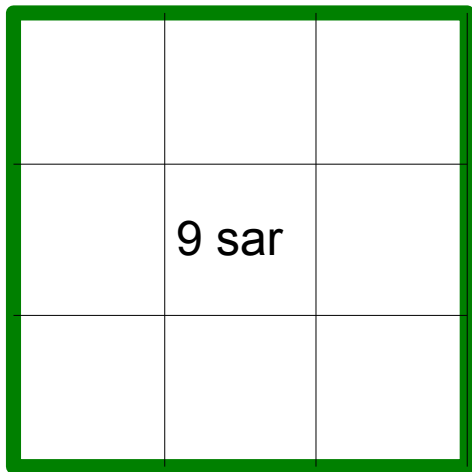
	IKU	ikum	100 SAR	circa 3600 mq
--	-----	------	---------	---------------

	BUR	burum	18 IKU	circa 64800
--	-----	-------	--------	-------------

	SHAR		60 BUR	circa 3.900.000 mq
--	------	--	--------	--------------------

mq

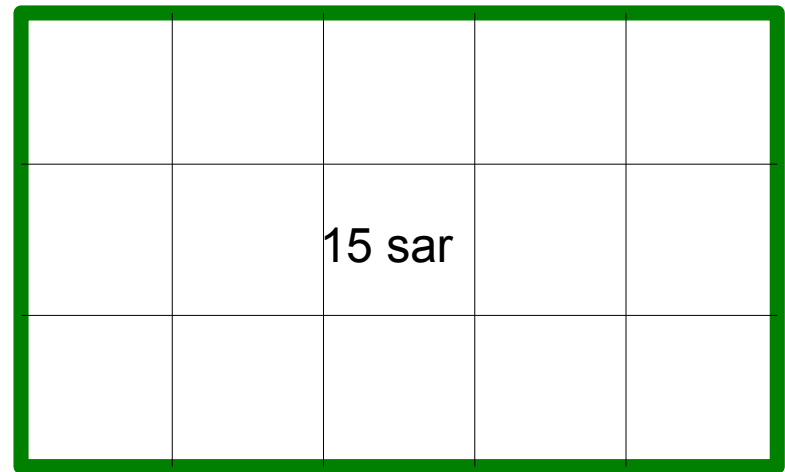
# QUADRATI e RETTANGOLI



3 ninda

$$A=|x|$$

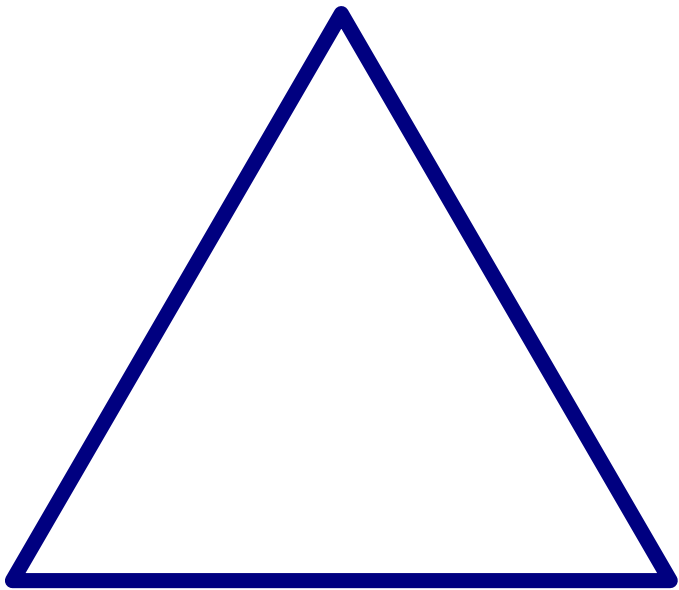
3 ninda



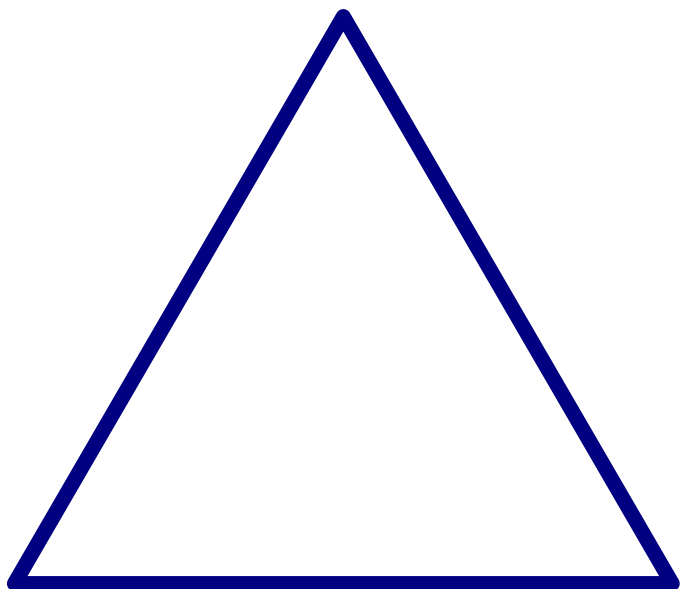
5 ninda

$$A=b \times h$$

TRIANGOLO o CUNEO?

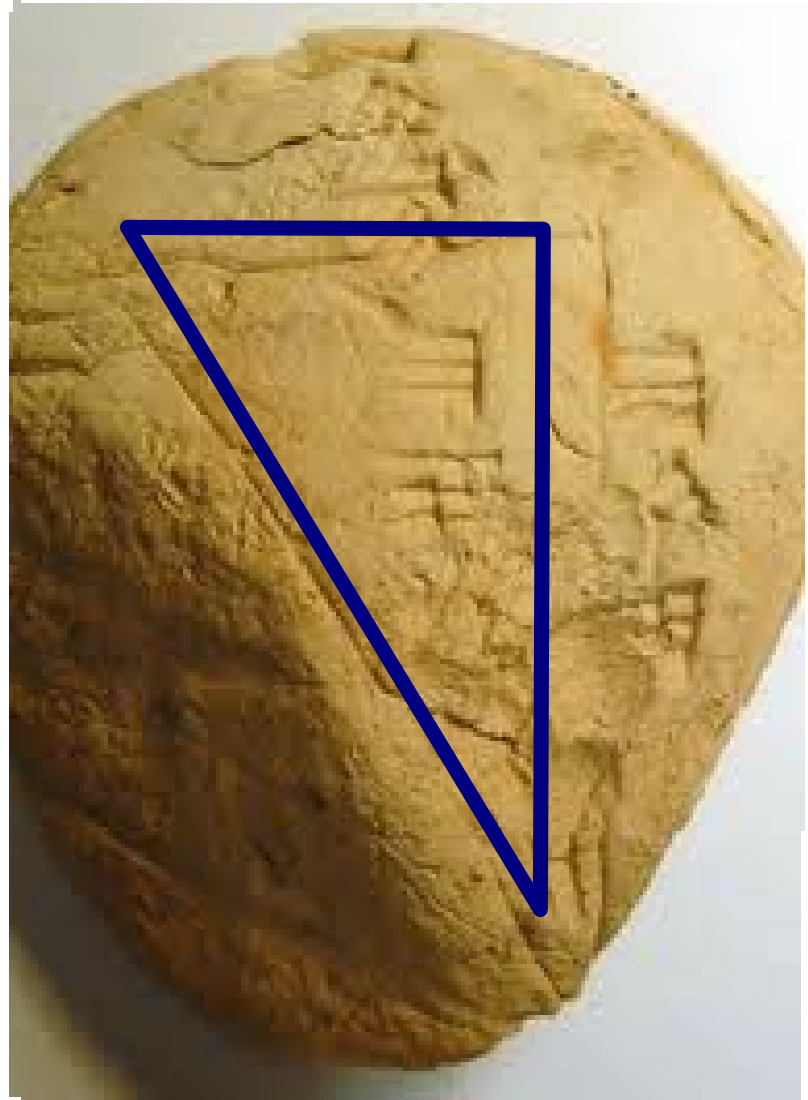
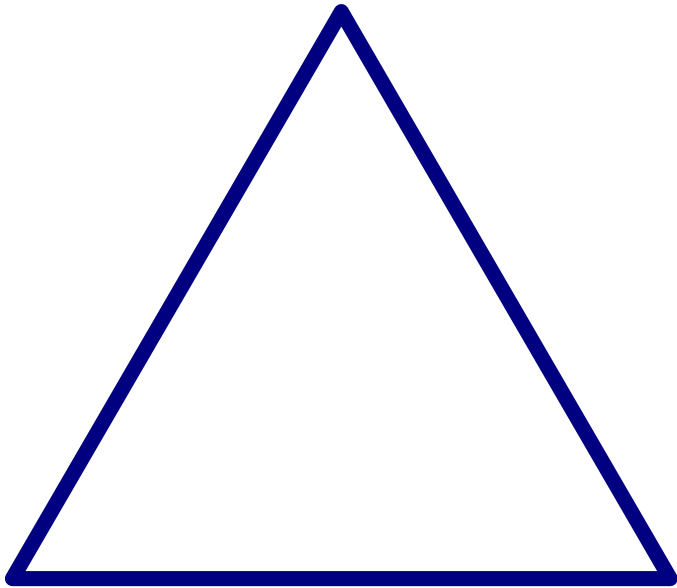


TRIANGOLO o CUNEO?

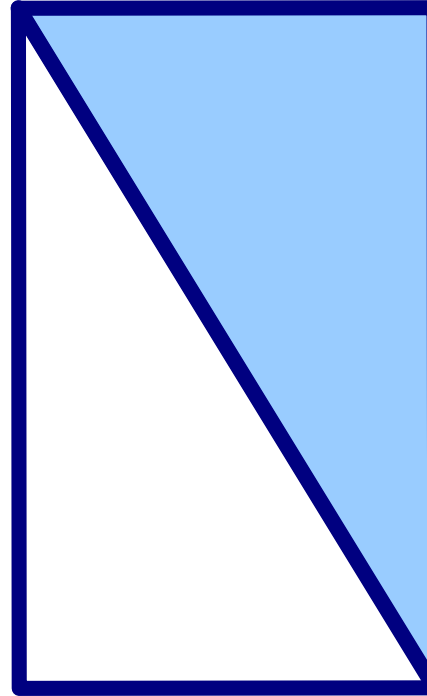
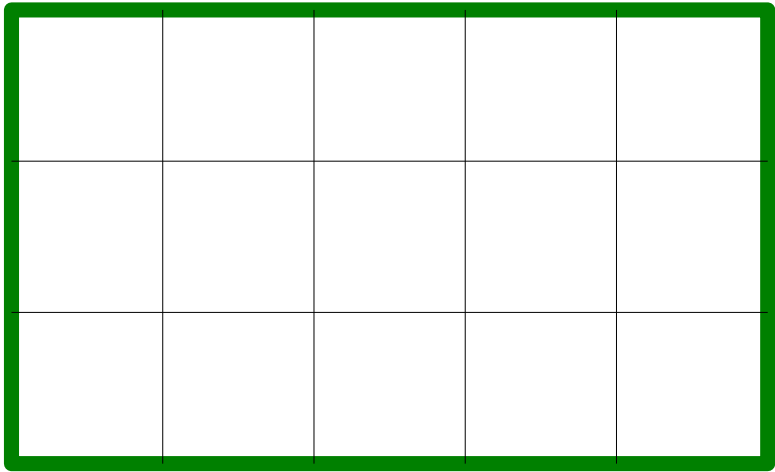




TRIANGOLO o CUNEO?

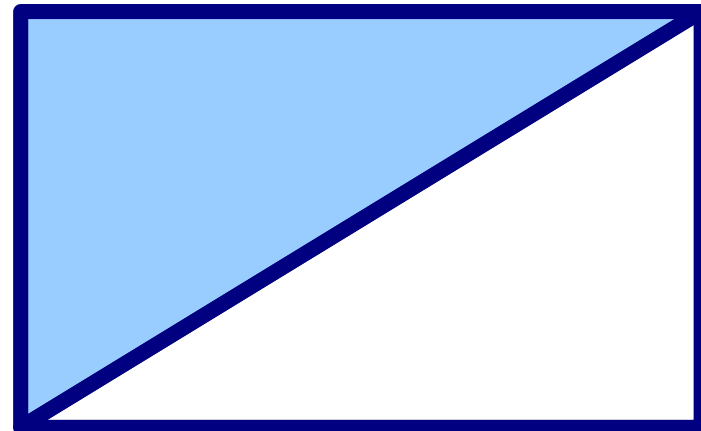


TRIANGOLO o CUNEO?

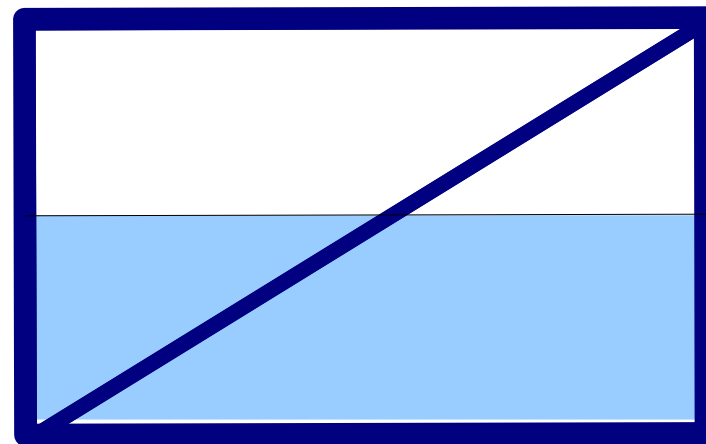
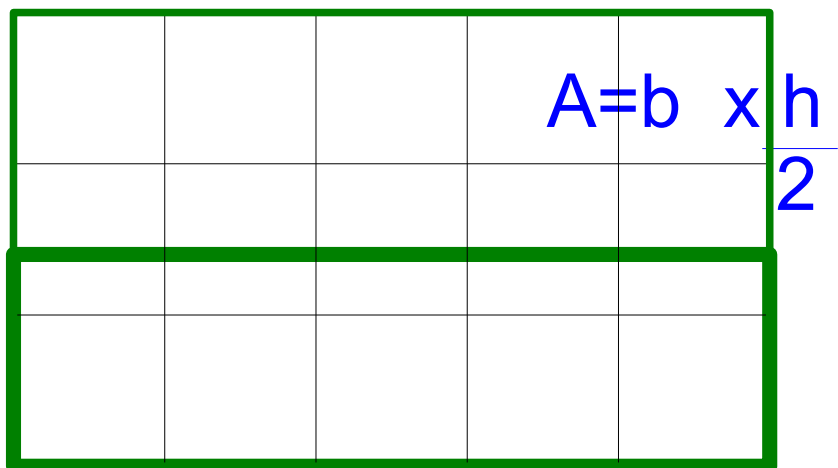
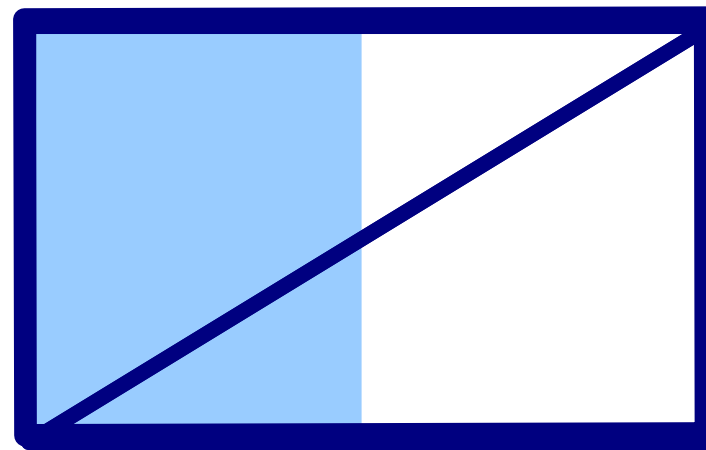
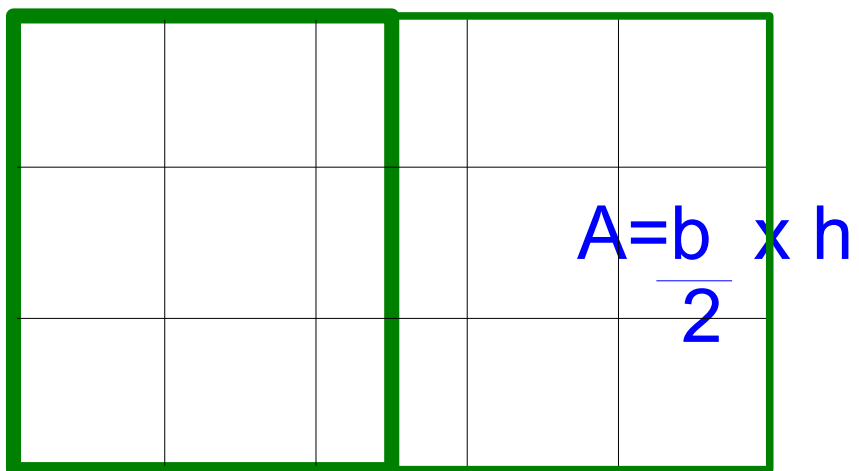


$$A=b \times h$$

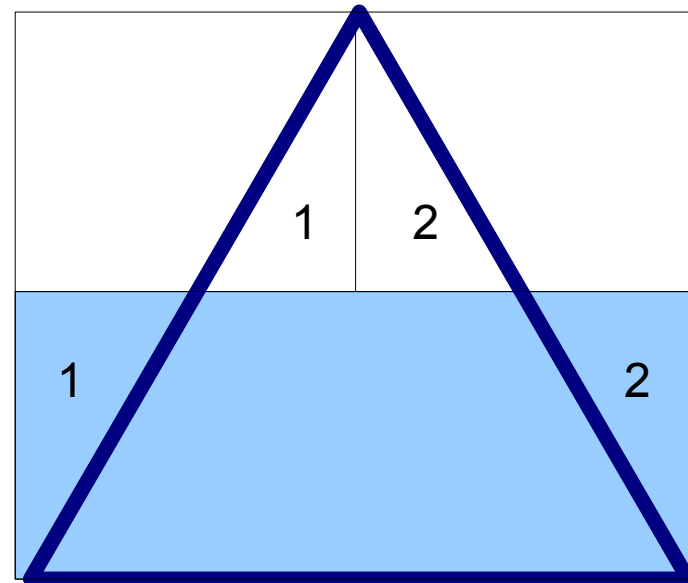
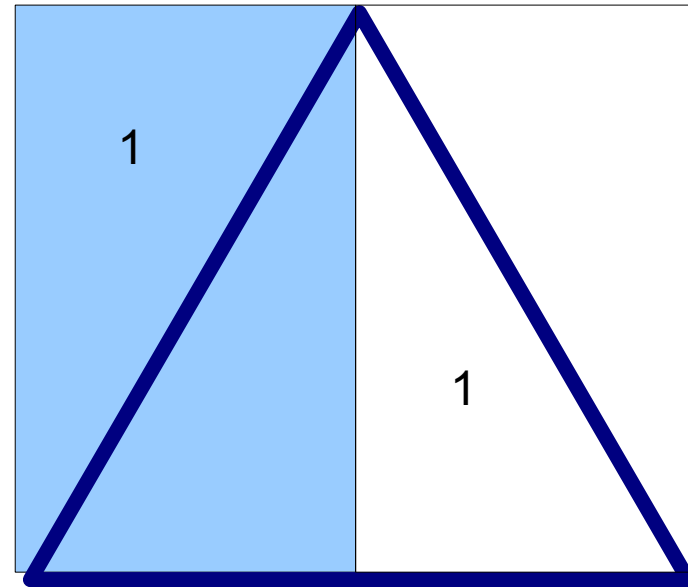
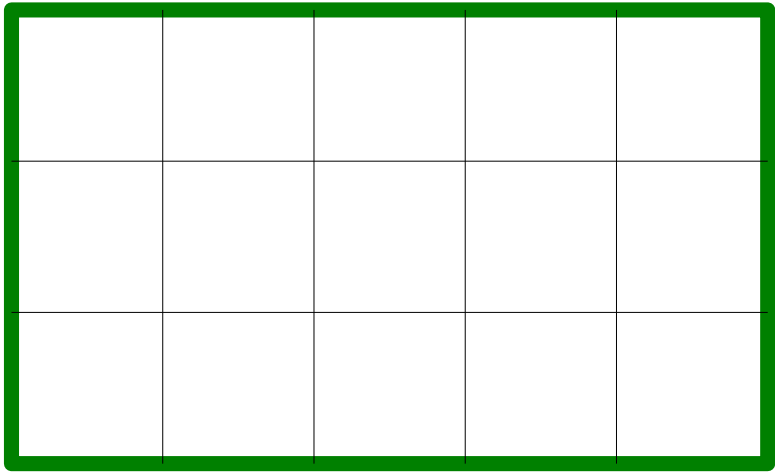
$$A=(b \times h)/2$$



TRIANGOLO o CUNEO?



TRIANGOLO o CUNEO?

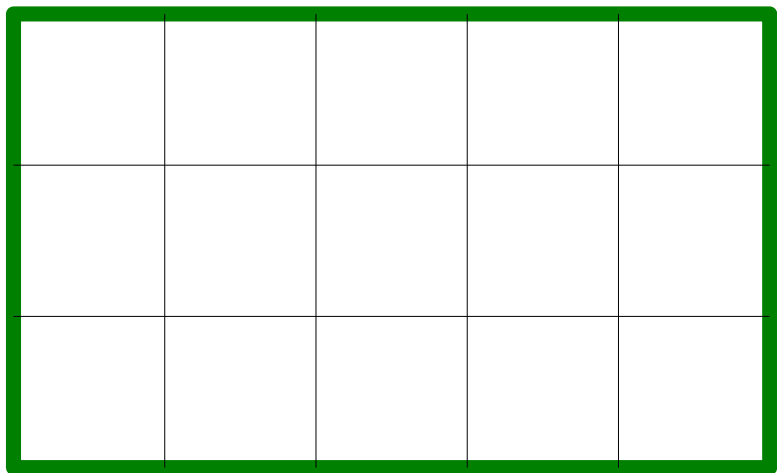


$$A = (b \times h) / 2$$

$$A = \frac{b}{2} \times h$$

$$A = b \times \frac{h}{2}$$

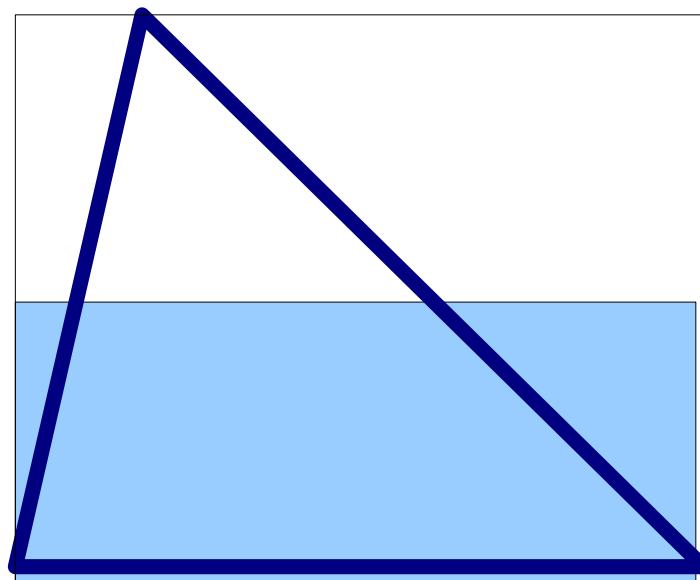
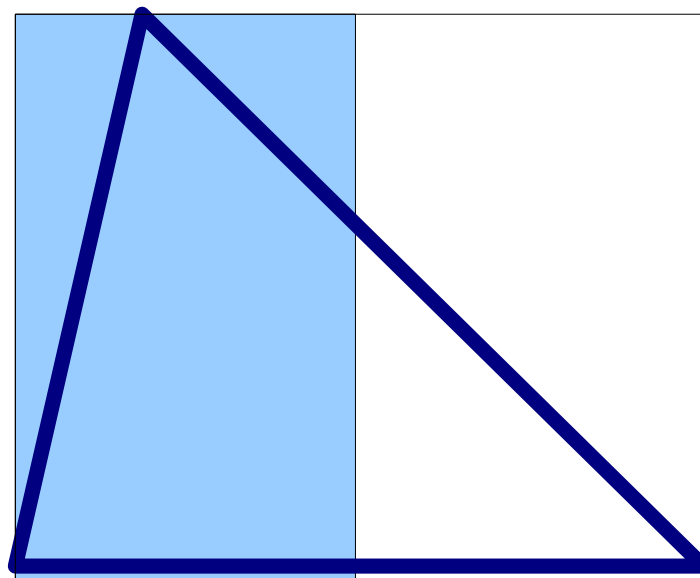
TRIANGOLO o CUNEO?



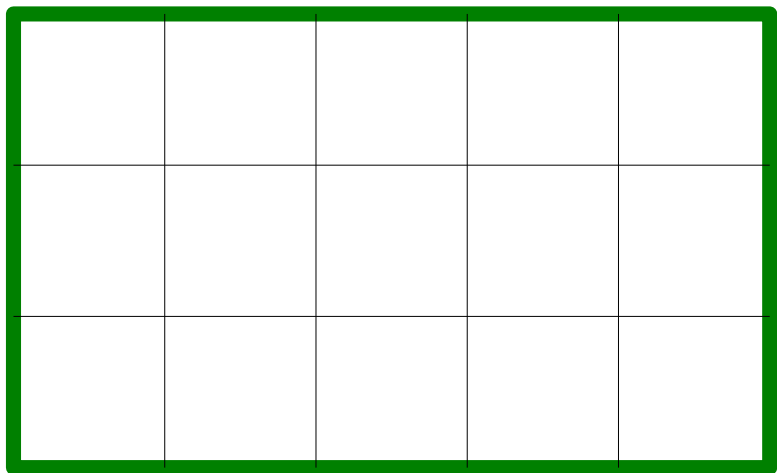
$$A = (b \times h) / 2$$

$$A = \frac{b \times h}{2}$$

$$A = b \times \frac{h}{2}$$



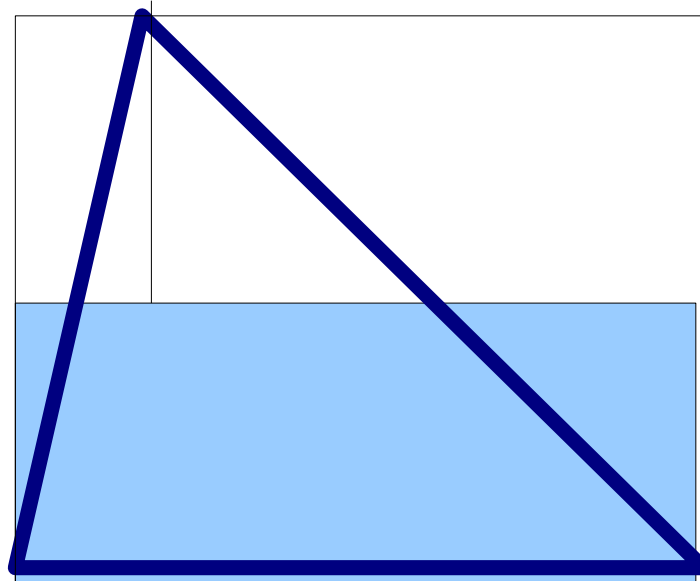
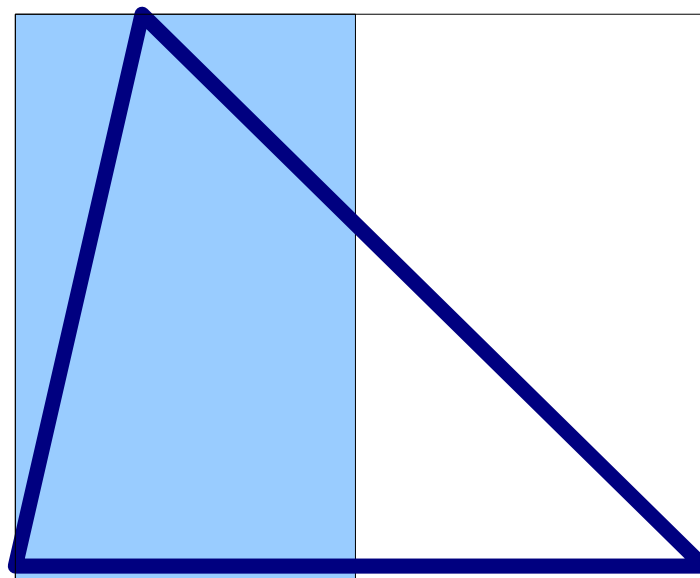
TRIANGOLO o CUNEO?



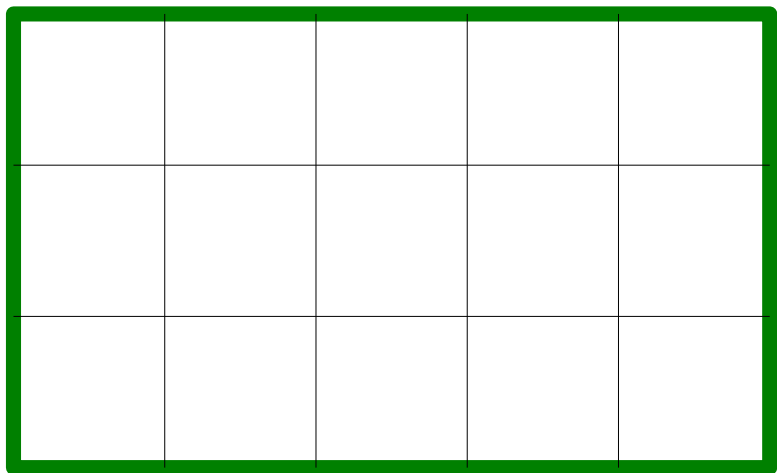
$$A = (b \times h) / 2$$

$$A = \frac{b \times h}{2}$$

$$A = b \times \frac{h}{2}$$



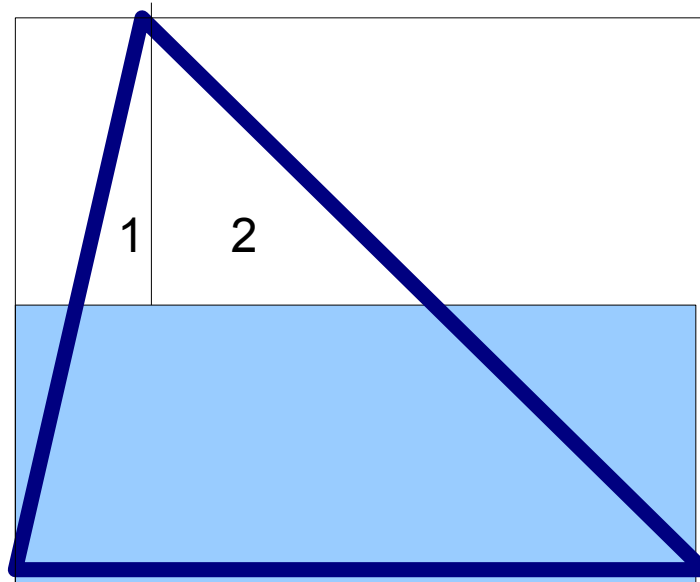
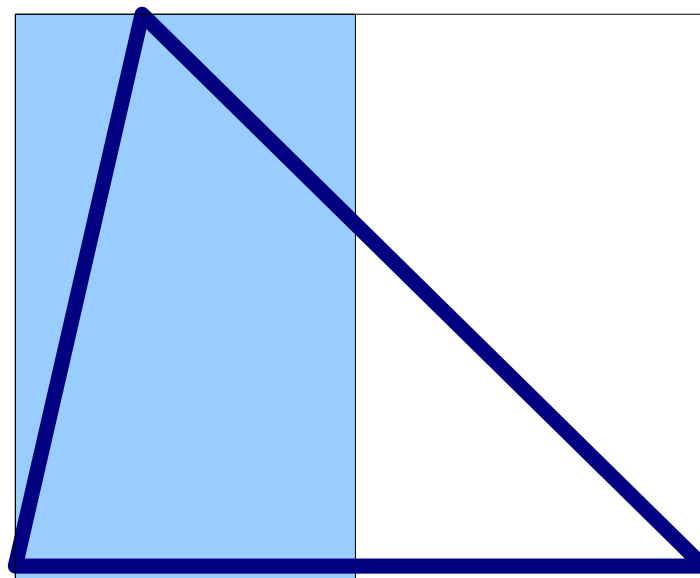
TRIANGOLO o CUNEO?



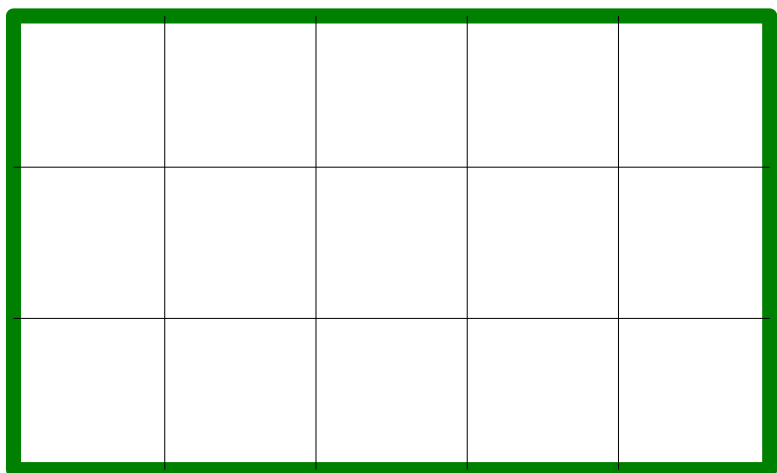
$$A = (b \times h) / 2$$

$$A = \frac{b \times h}{2}$$

$$A = b \times \frac{h}{2}$$



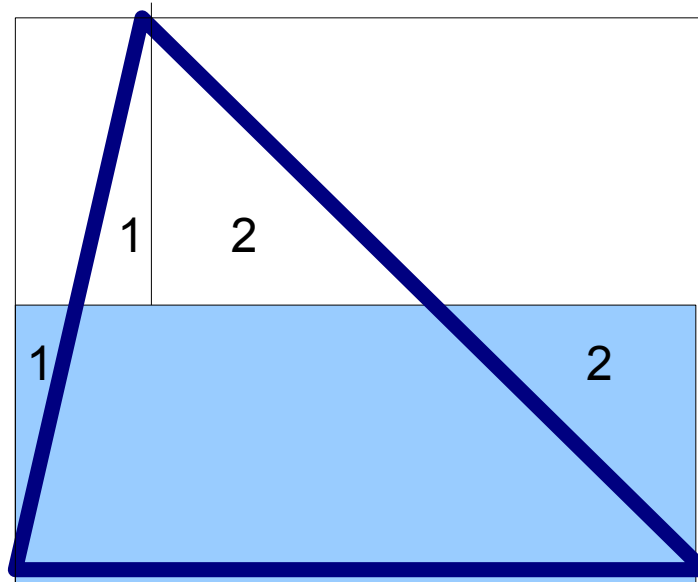
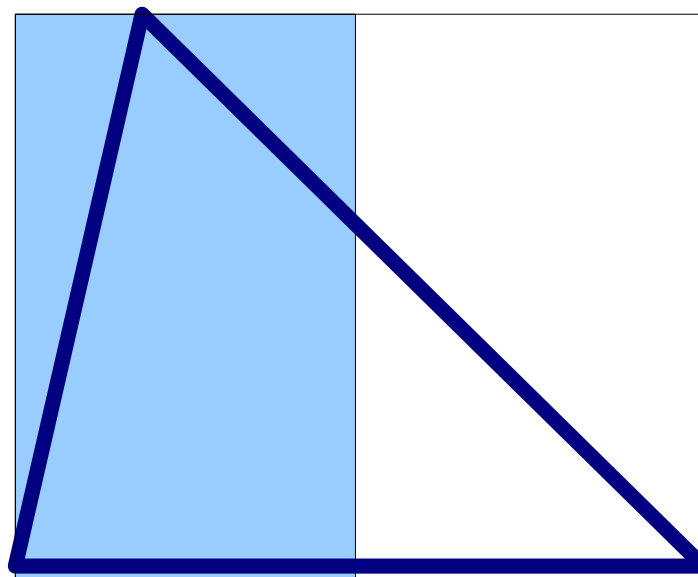
TRIANGOLO o CUNEO?



$$A = (b \times h) / 2$$

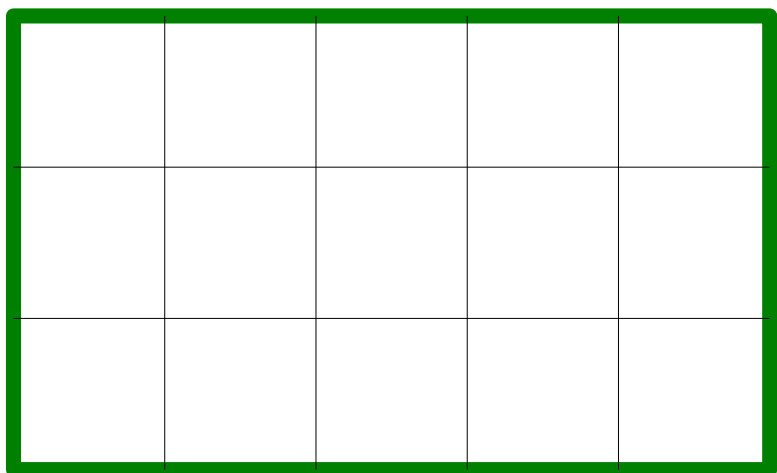
$$A = \frac{b \times h}{2}$$

$$A = b \times \frac{h}{2}$$





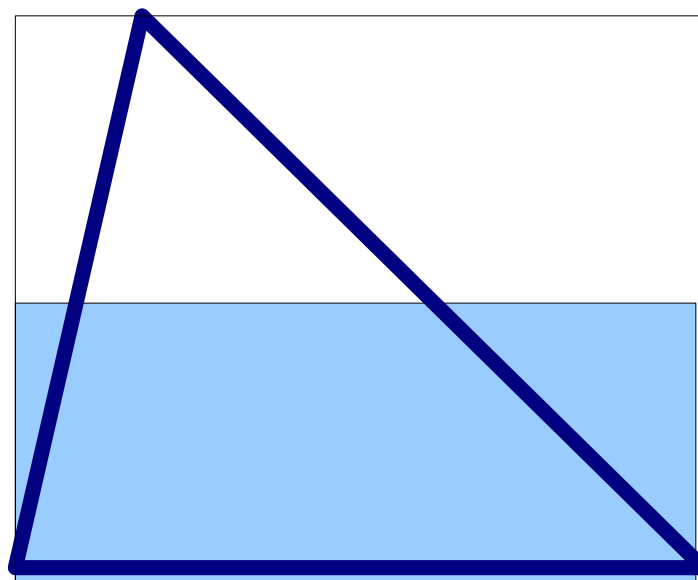
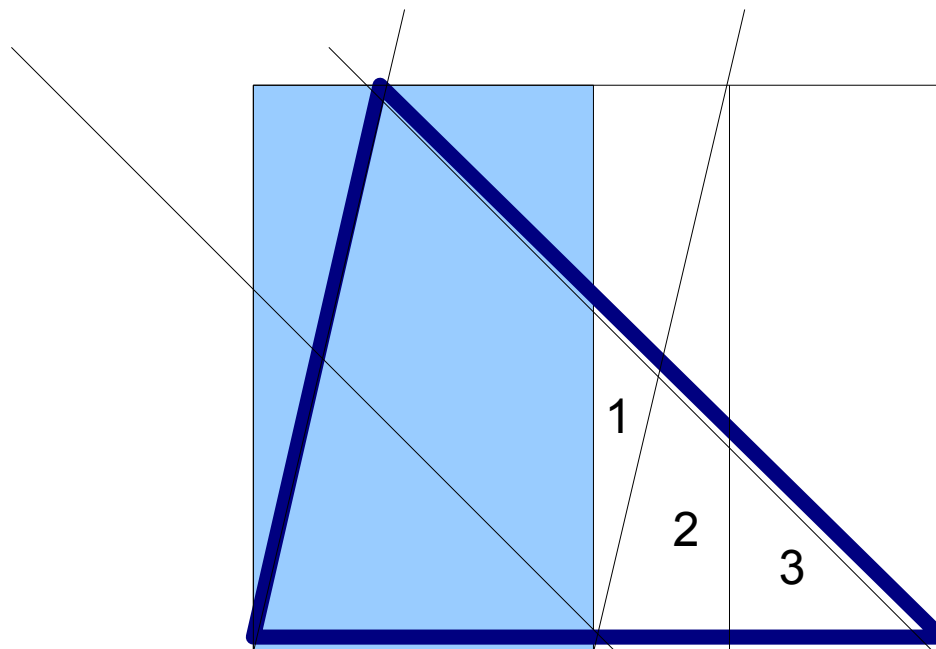
TRIANGOLO o CUNEO?



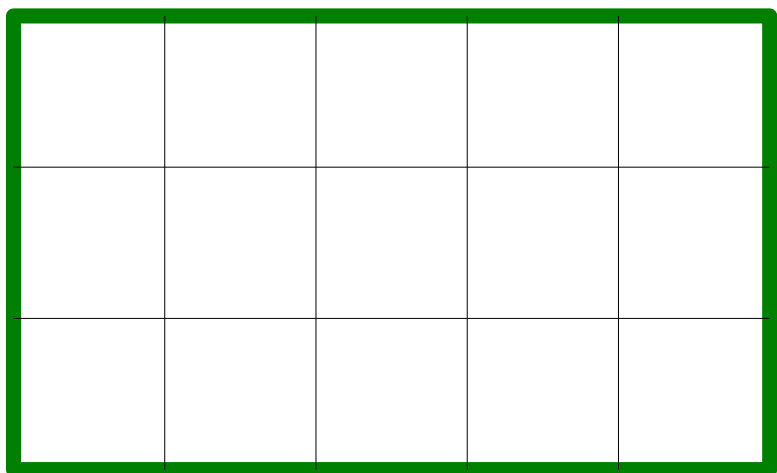
$$A = (b \times h) / 2$$

$$A = \frac{b \times h}{2}$$

$$A = b \times \frac{h}{2}$$



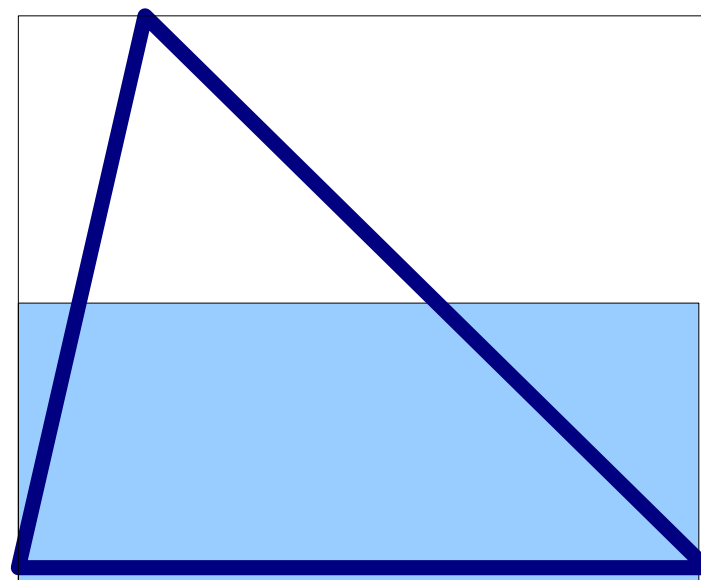
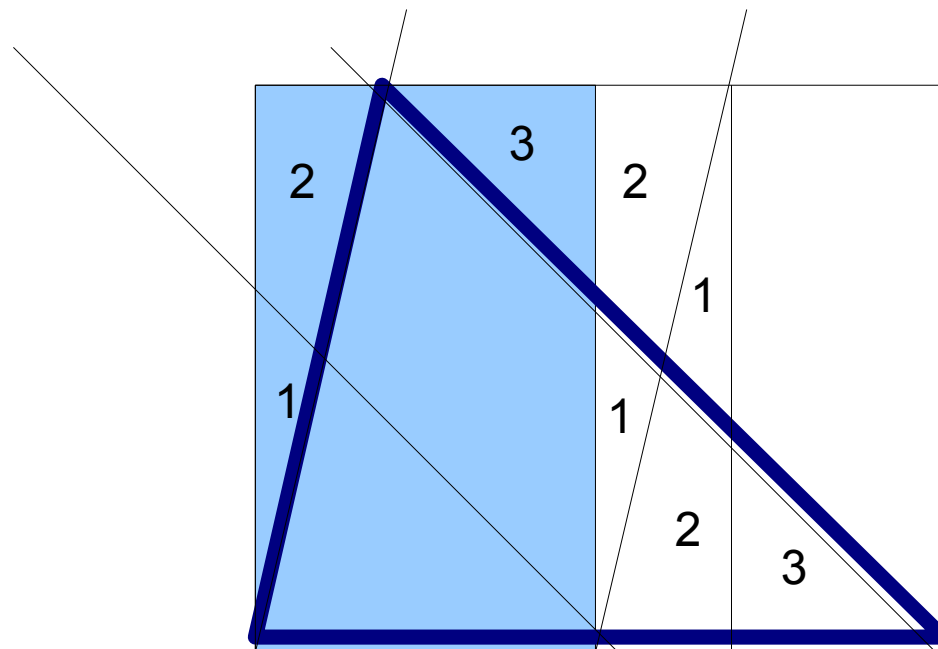
TRIANGOLO o CUNEO?



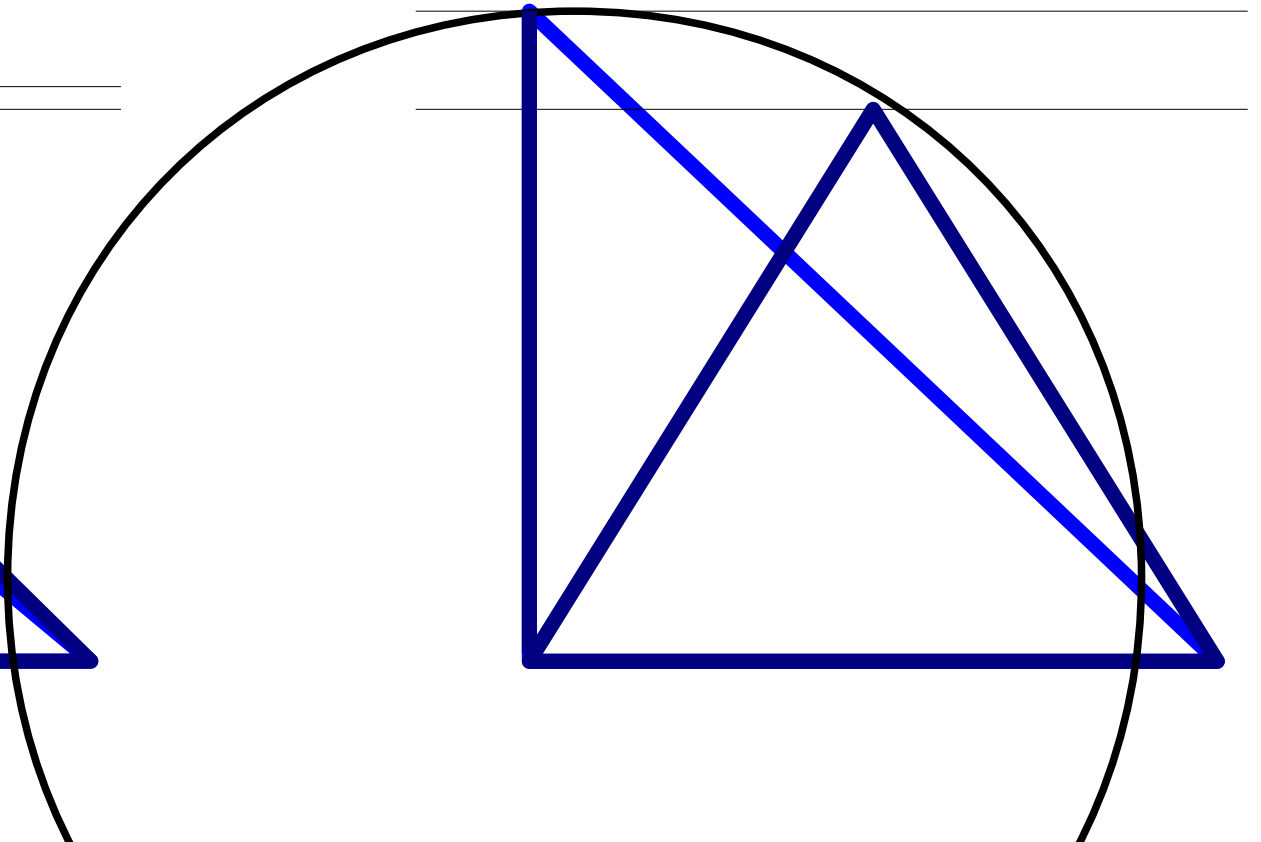
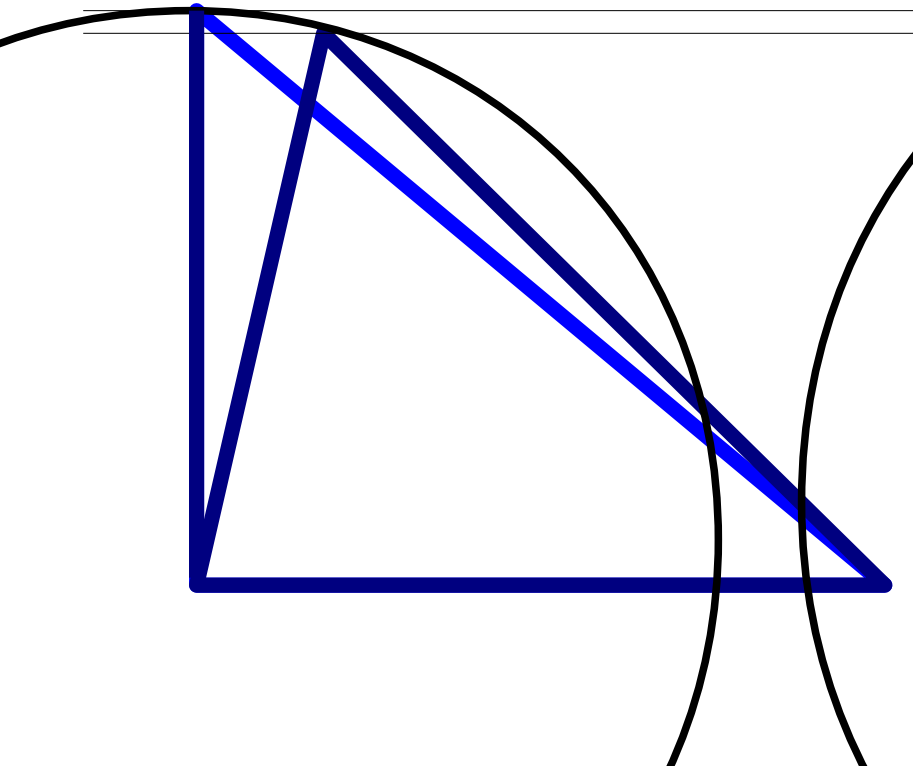
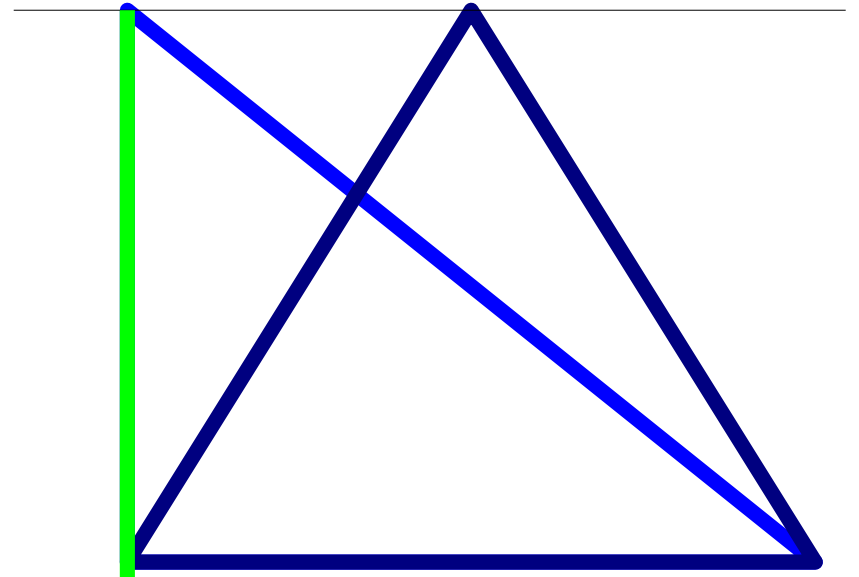
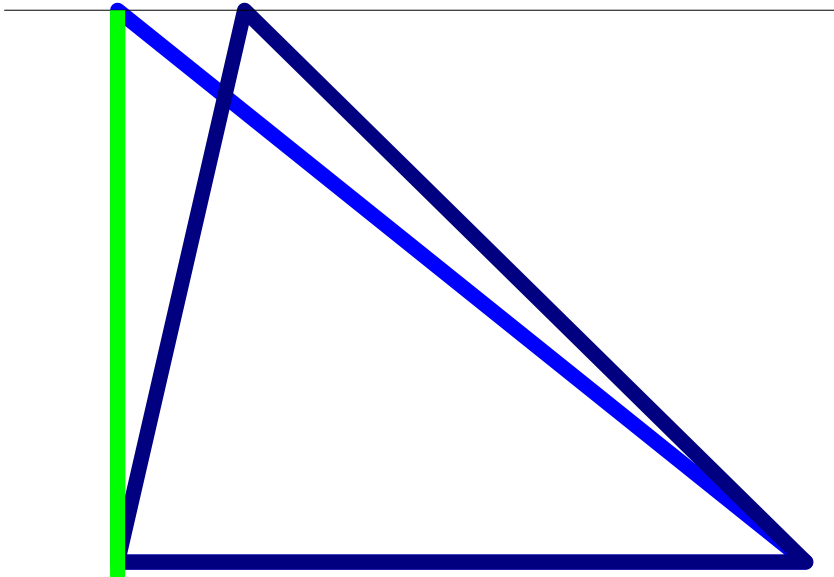
$$A = (b \times h) / 2$$

$$A = \frac{b \times h}{2}$$

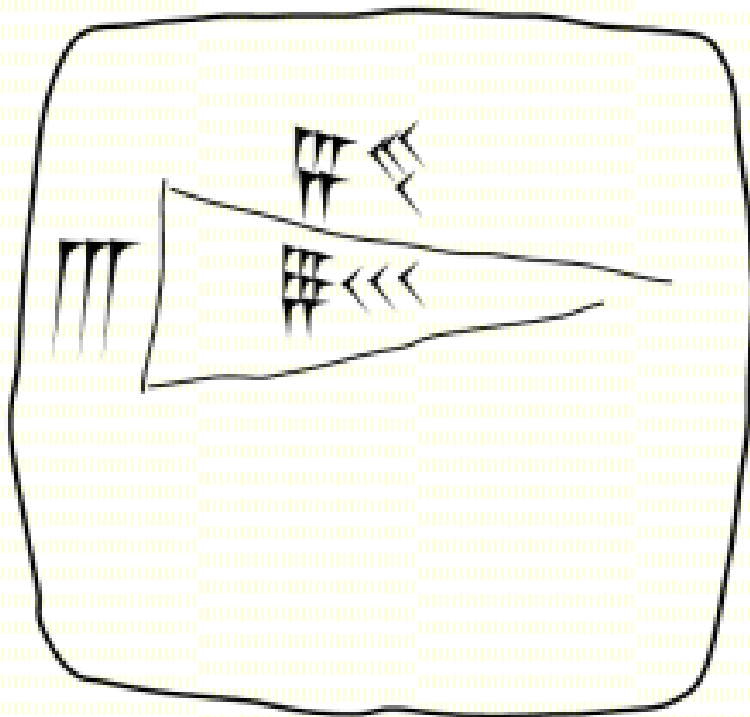
$$A = b \times \frac{h}{2}$$



ALTEZZA o LUNGHEZZA?



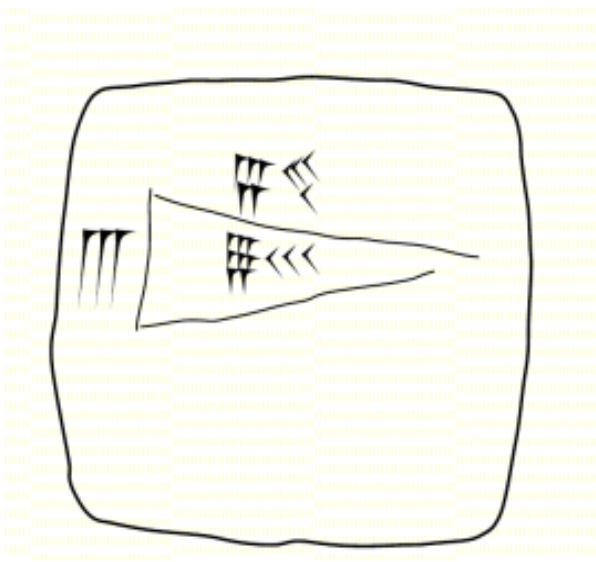
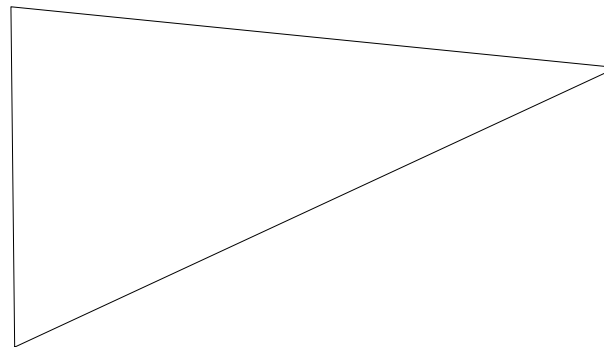
L'area di un triangolo



Verso: vuoto

MS 3042 Schoyen collection

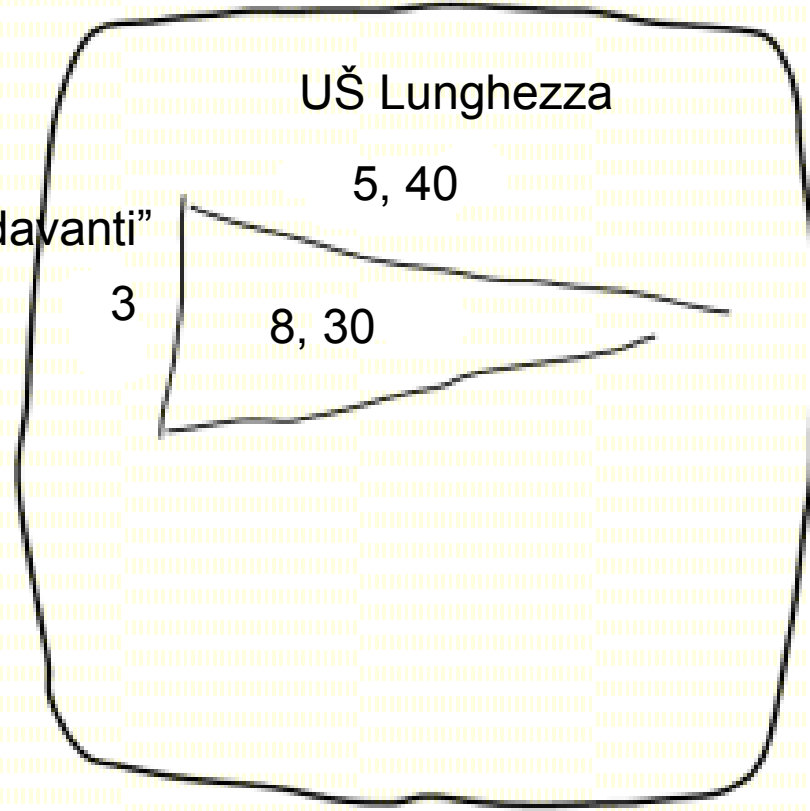
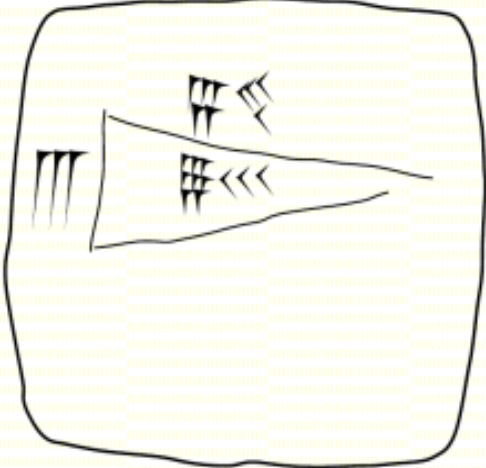
L'area di un triangolo



MS 3042 Schoyen collection

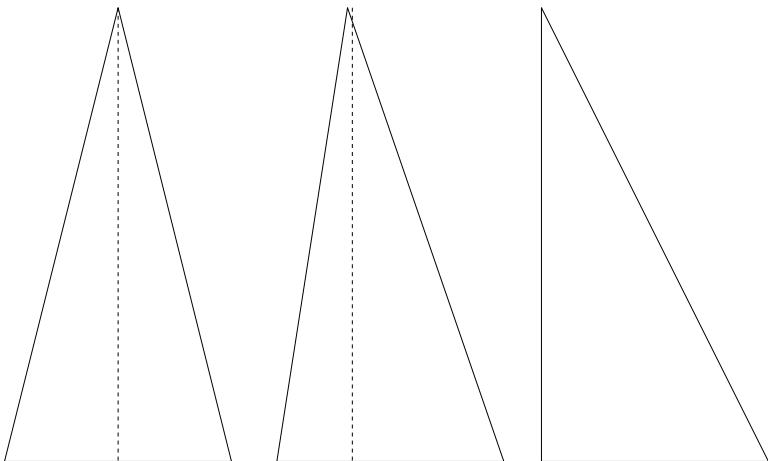
L'area di un triangolo

SAG "davanti"



Osservazioni  
testo scolastico di insegnamento elementare

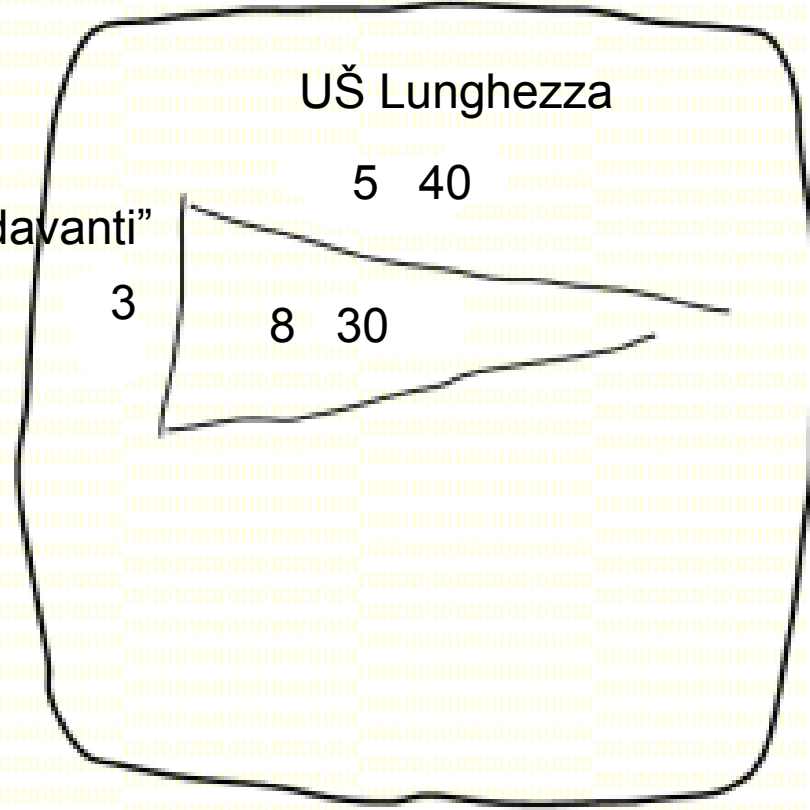
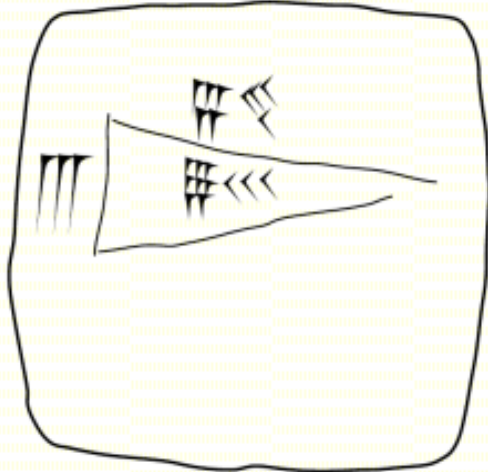
- non si distingue tra "altezza" e "lato lungo"



MS 3042 Schoyen collection

L'area di un triangolo

SAG "davanti"



Unità di misura

lunghezza ninda (circa 6m)

area sar (ninda x ninda)  
= 36 mq circa

Osservazioni  
testo scolastico di insegnamento elementare

- interpretazione valori numerici

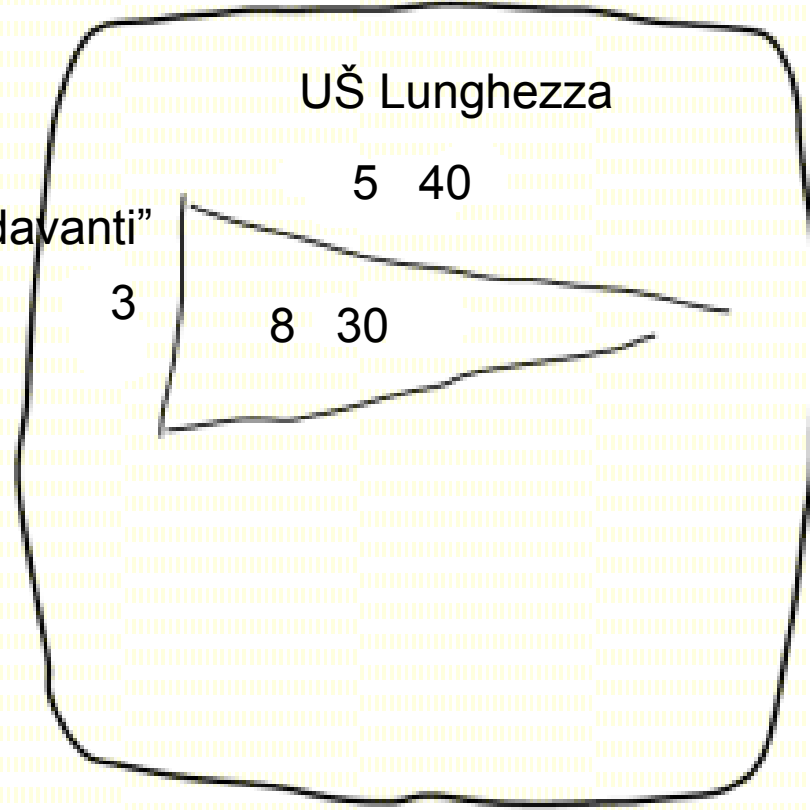
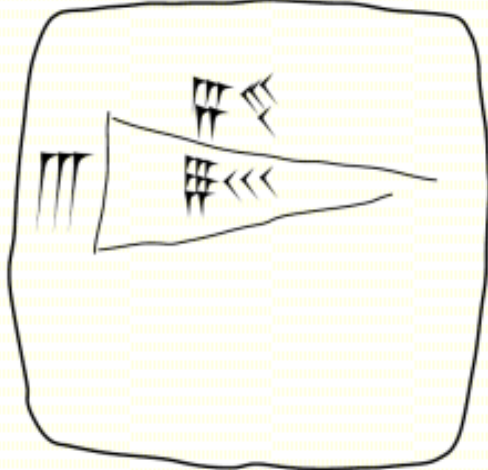
5 40 -----> 5, 40 ninda (notazione sessagesimale)  
 3 -----> 3, 00 ninda

In generale:  
misure corrispondenti a campi coltivati  
(problemi concreti ?)

MS 3042 Schoyen collection

L'area di un triangolo

SAG "davanti"



Osservazioni  
testo scolastico di insegnamento elementare

- interpretazione valori numerici

Unità di misura

lunghezza      ninda (circa 6m)

area              sar (ninda x ninda)  
= 36 mq circa  
iku (1, 40 ninda x ninda)

5 40 -----> 5, 40 ninda (notazione sessagesimale)  
3                -----> 3, 00 ninda

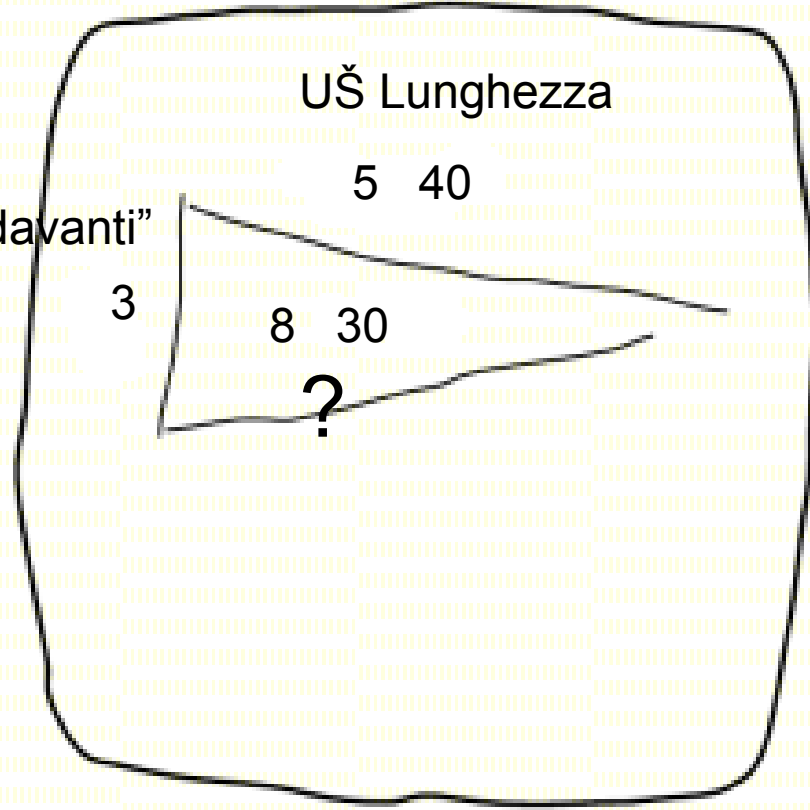
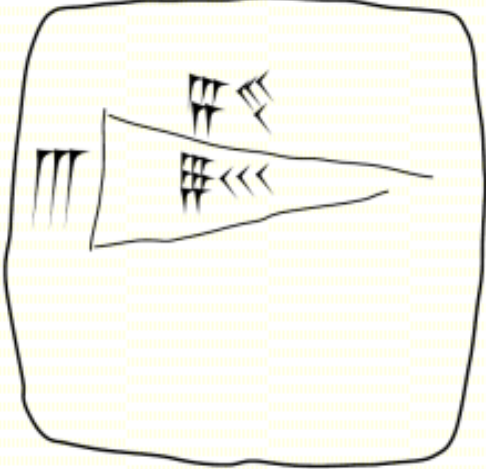




MS 3042 Schoyen collection

L'area di un triangolo

SAG "davanti"






$$5\ 40 \times 3\ 00 / 2$$

$$5\ 40 \times 3\ 00 \times 30$$

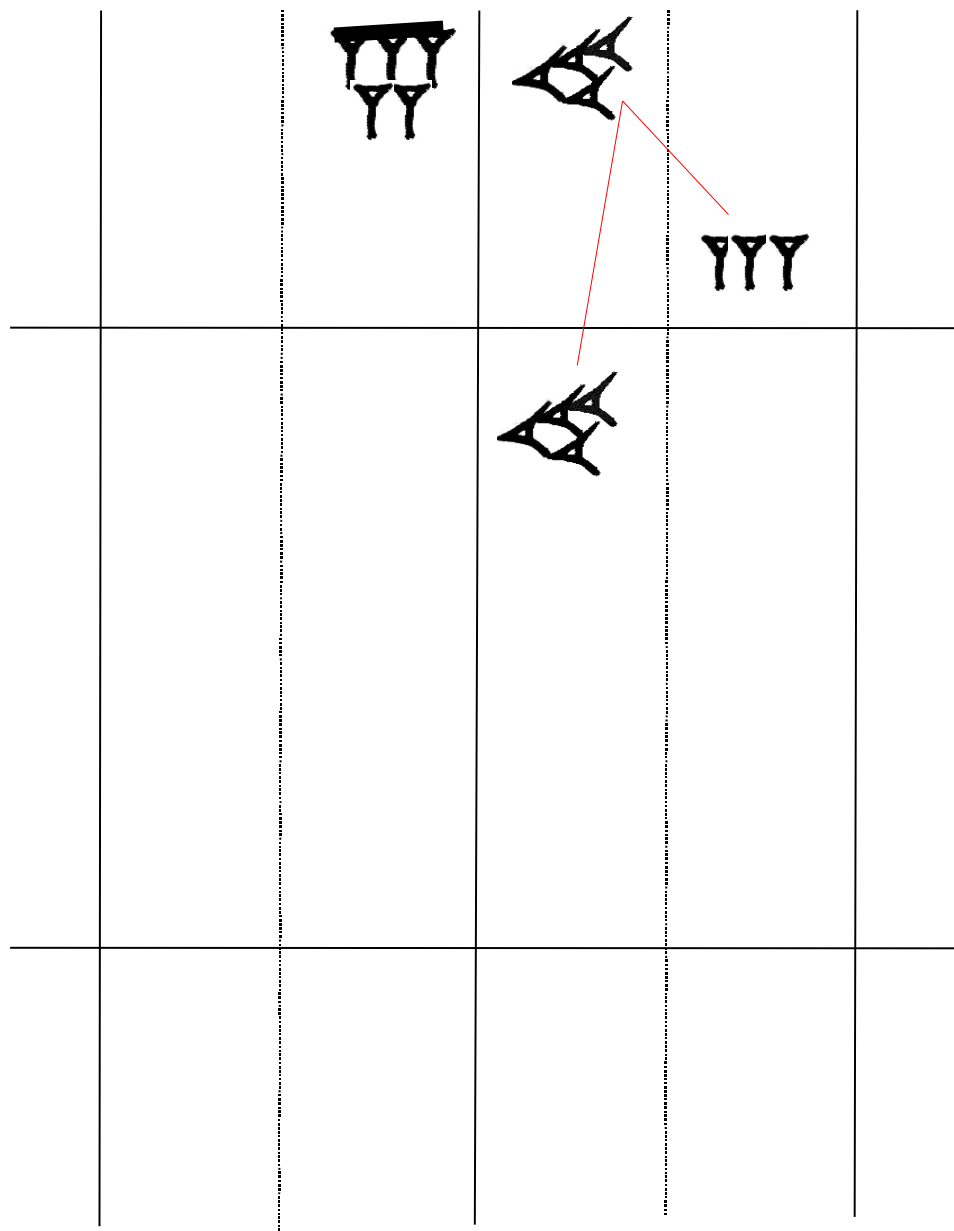
$$A=?$$

$$A=(b \times h) / 2$$

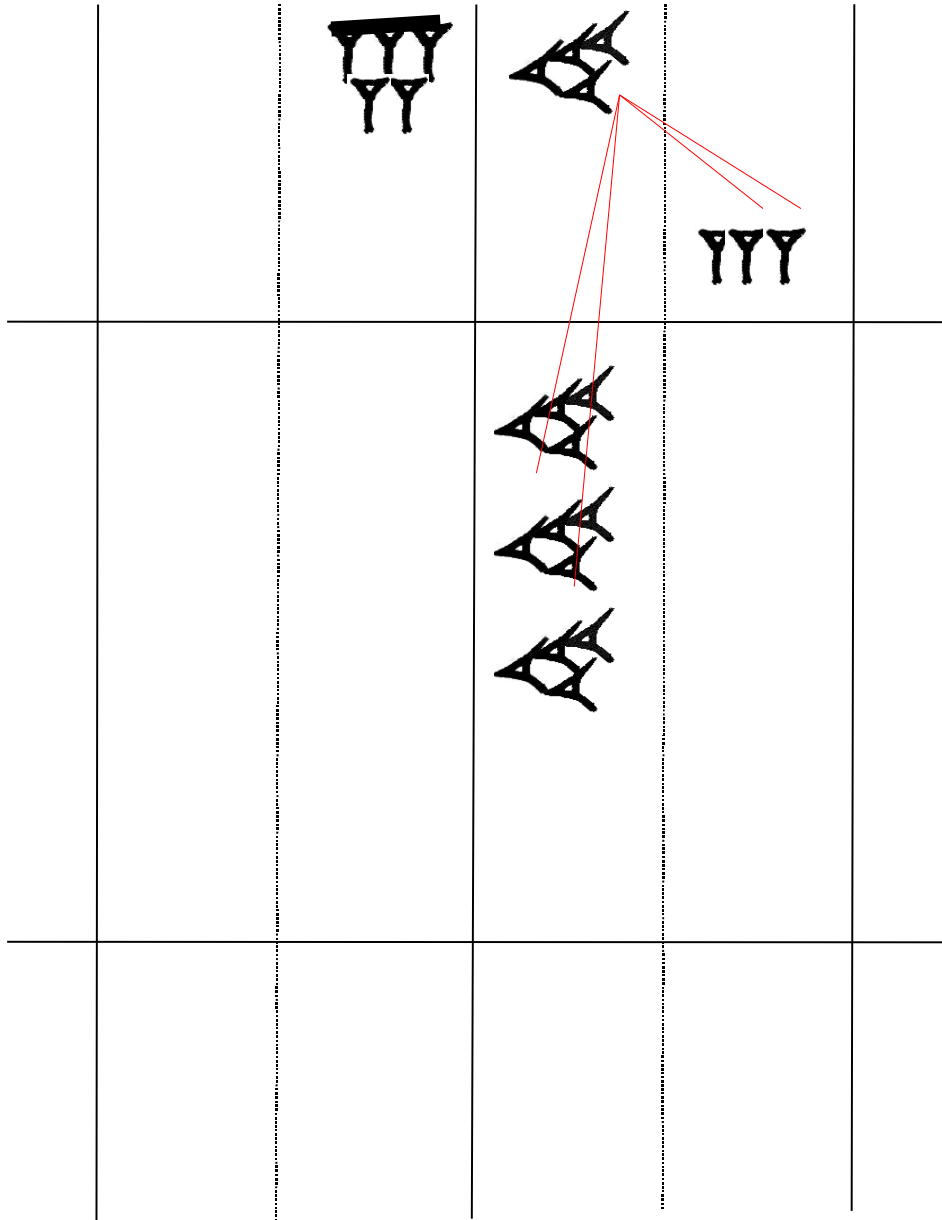
$$540 \times 3 = ?$$

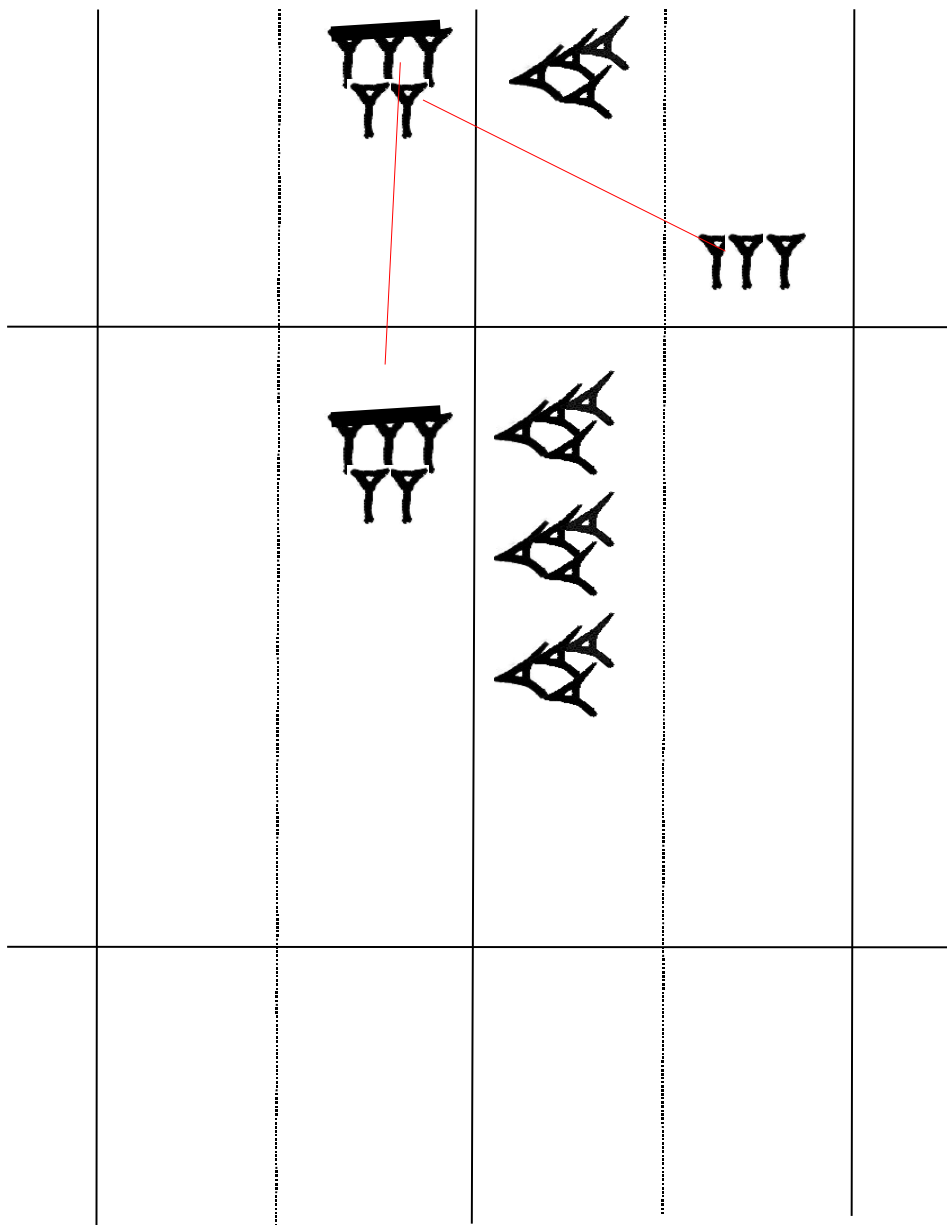
$$540 \times 3 = ?$$



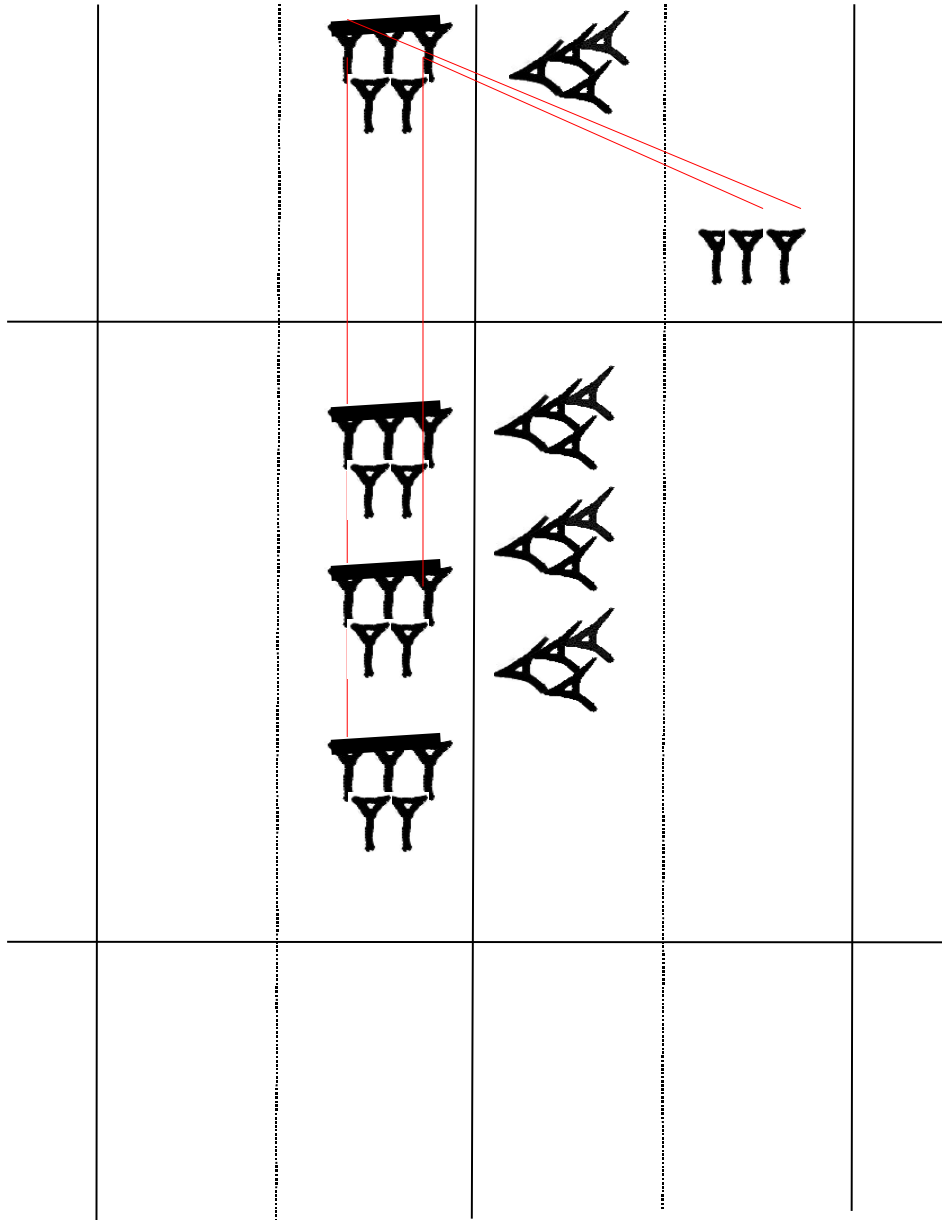
$$540 \times 3 = ?$$



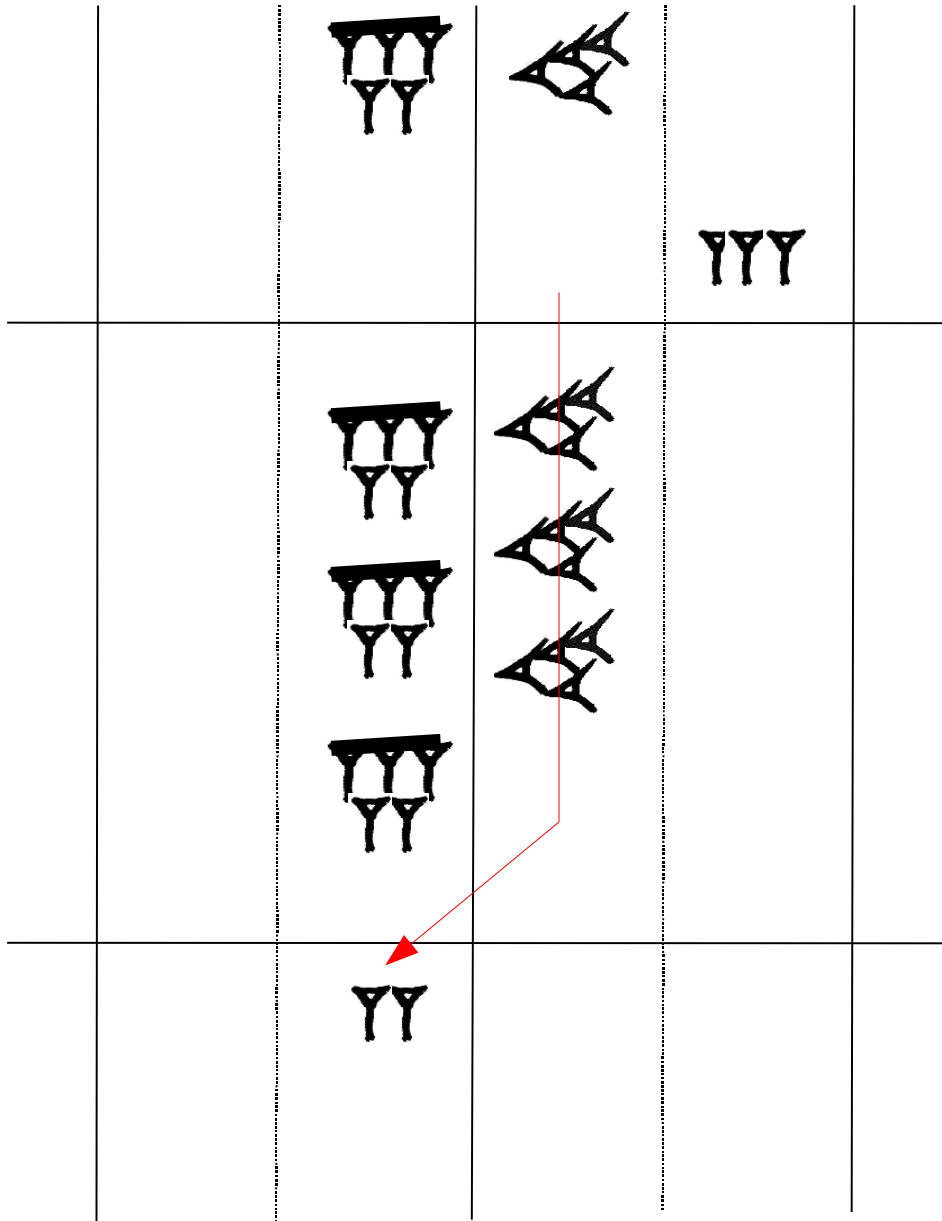
$$540 \times 3 = ?$$



$$540 \times 3 = ?$$

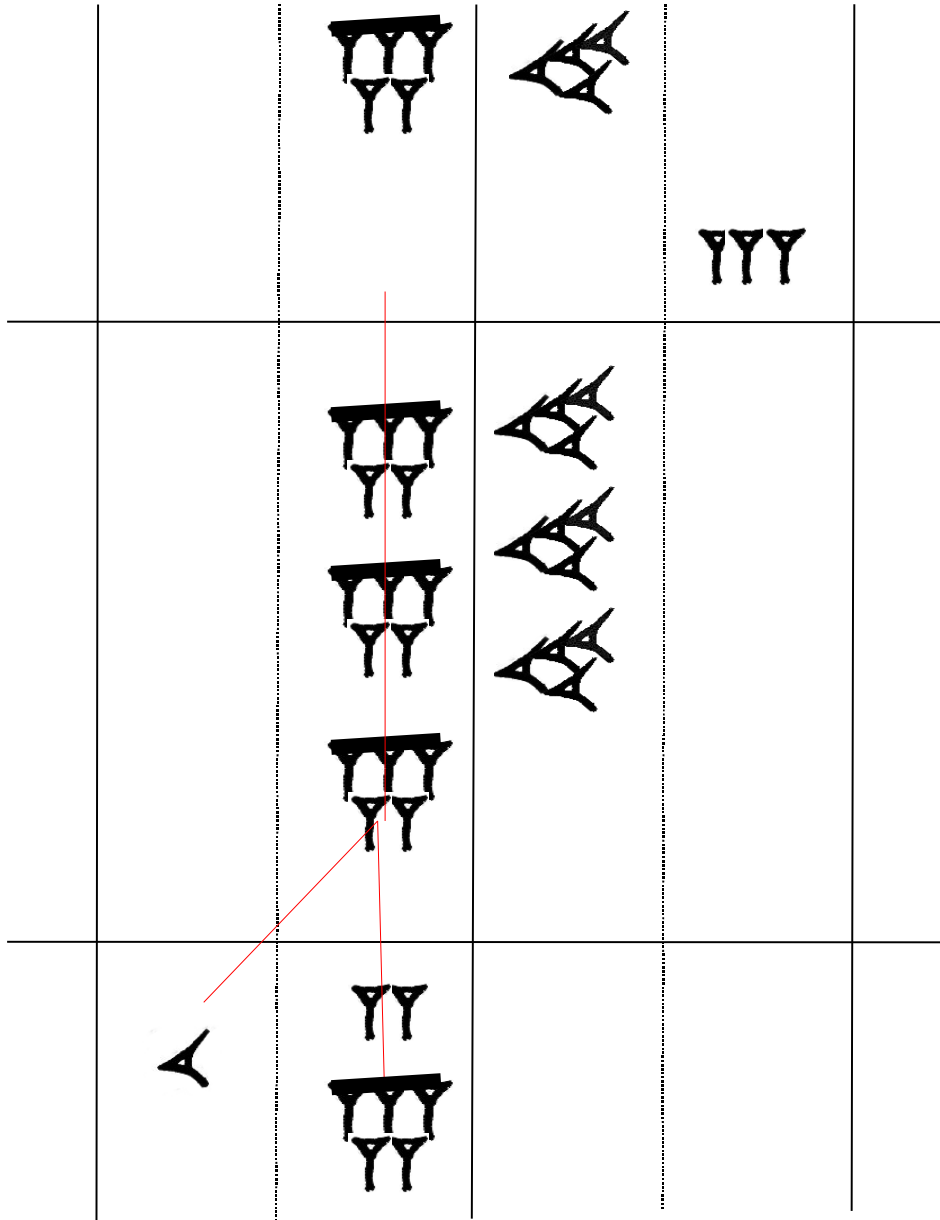


$$540 \times 3 = ?$$















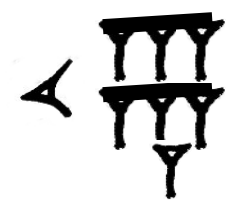


$$540 \times 3 = ?$$





$$540 \times 3 = 17$$

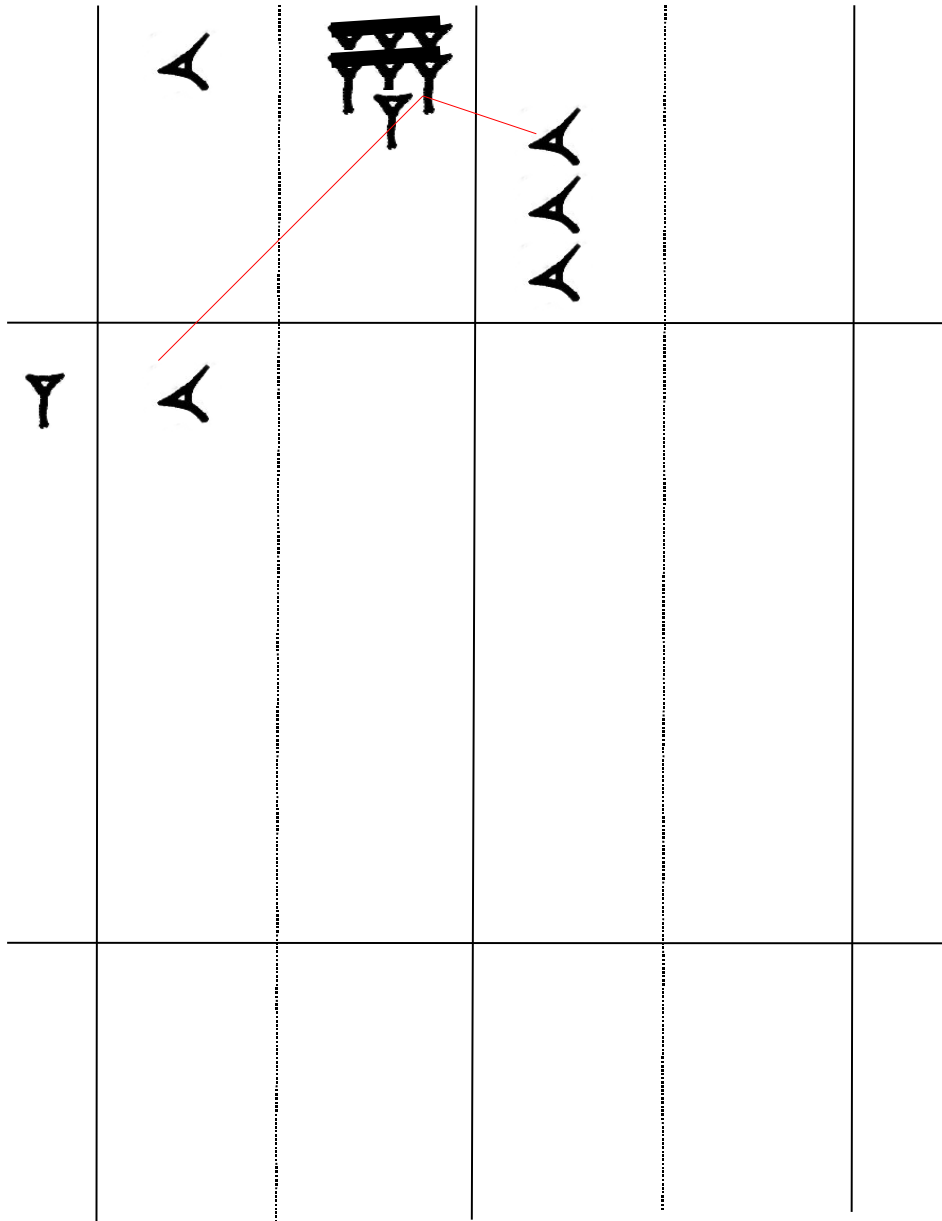
			
			
		  	  
	 		






$$1700 \times 0030 = ?$$

A		

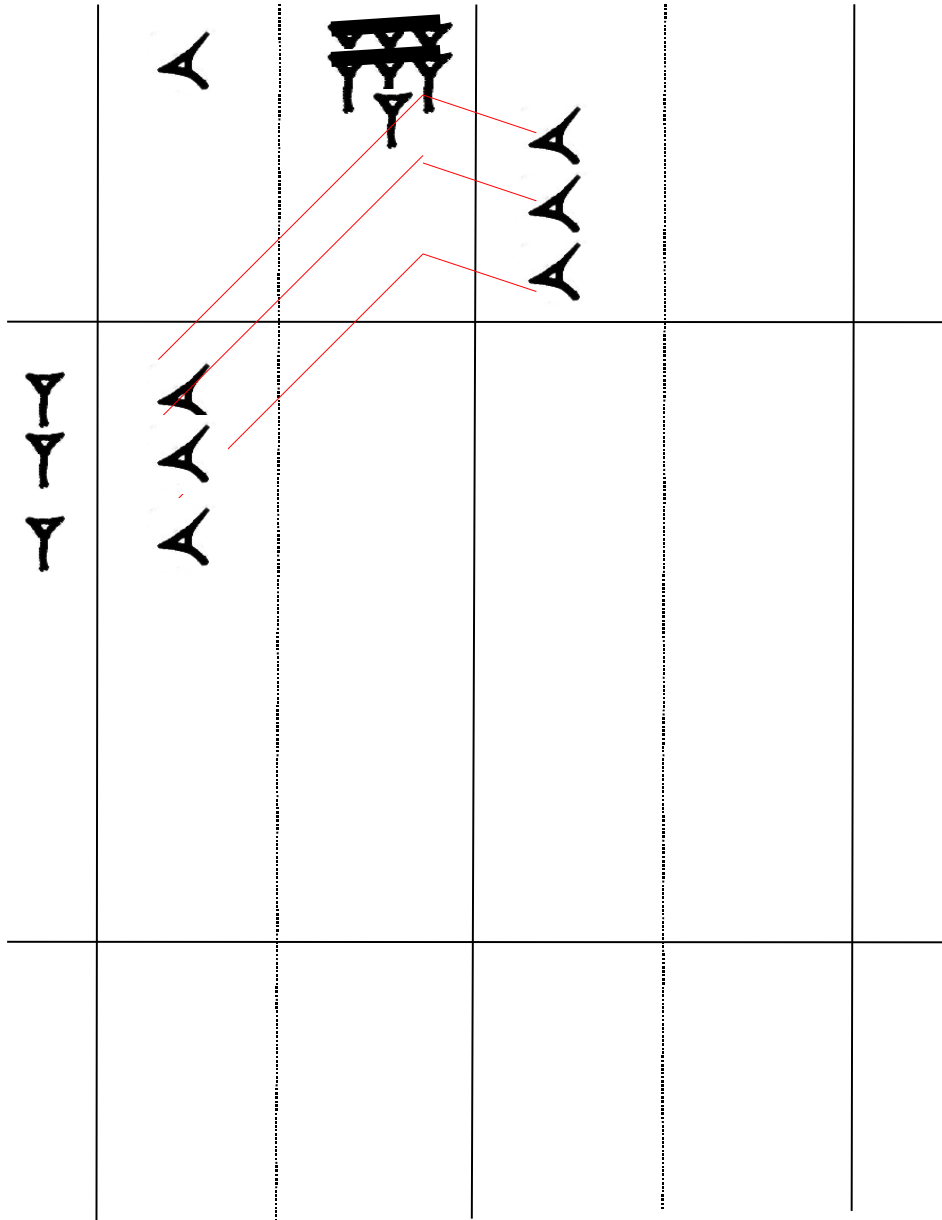
17 00 x 00 30 = ?



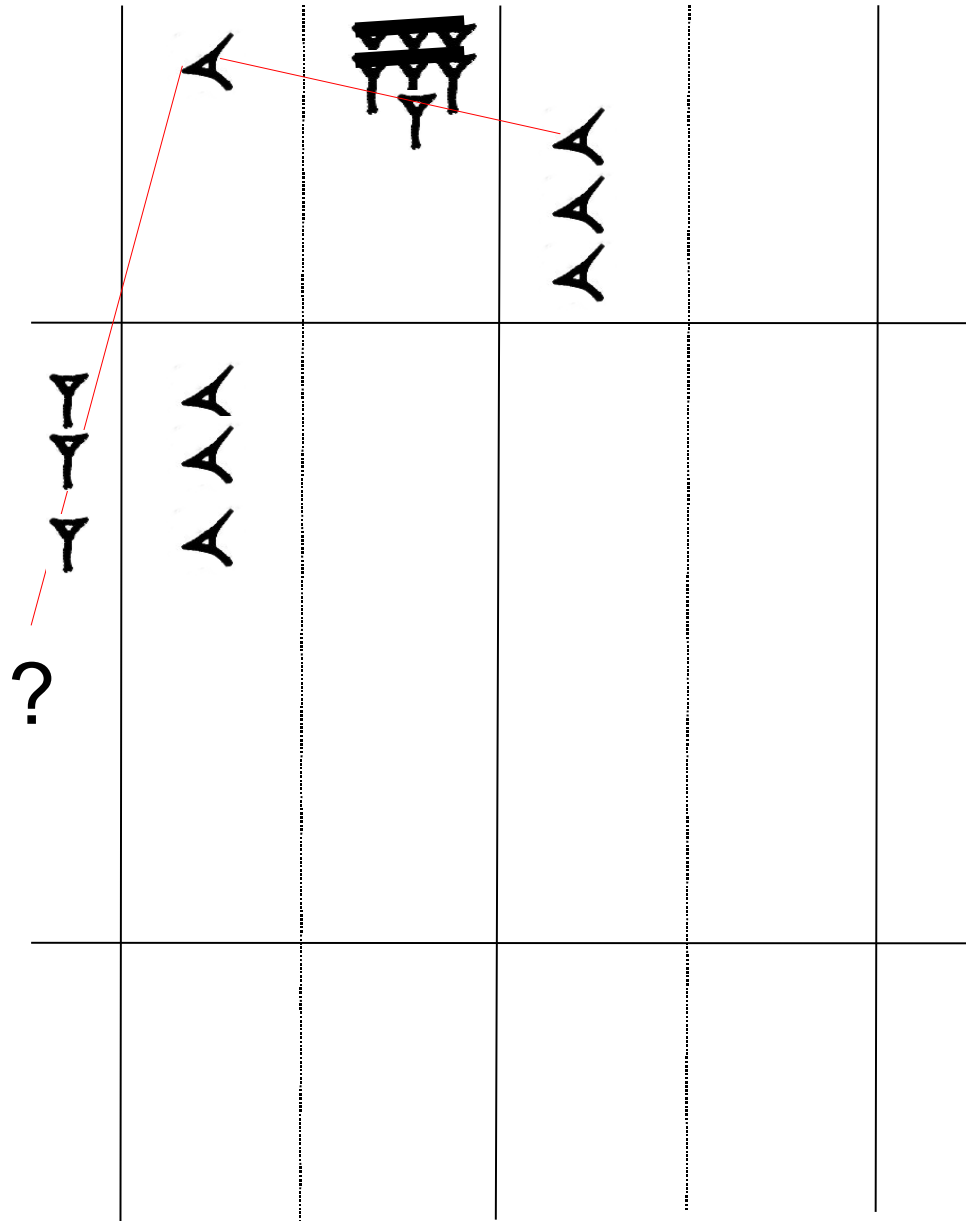
 x  =  

$7 \times 10 = 70$   
 $7 \times 10 = [1 \ 10]$

$$17\ 00 \times 00\ 30 = ?$$



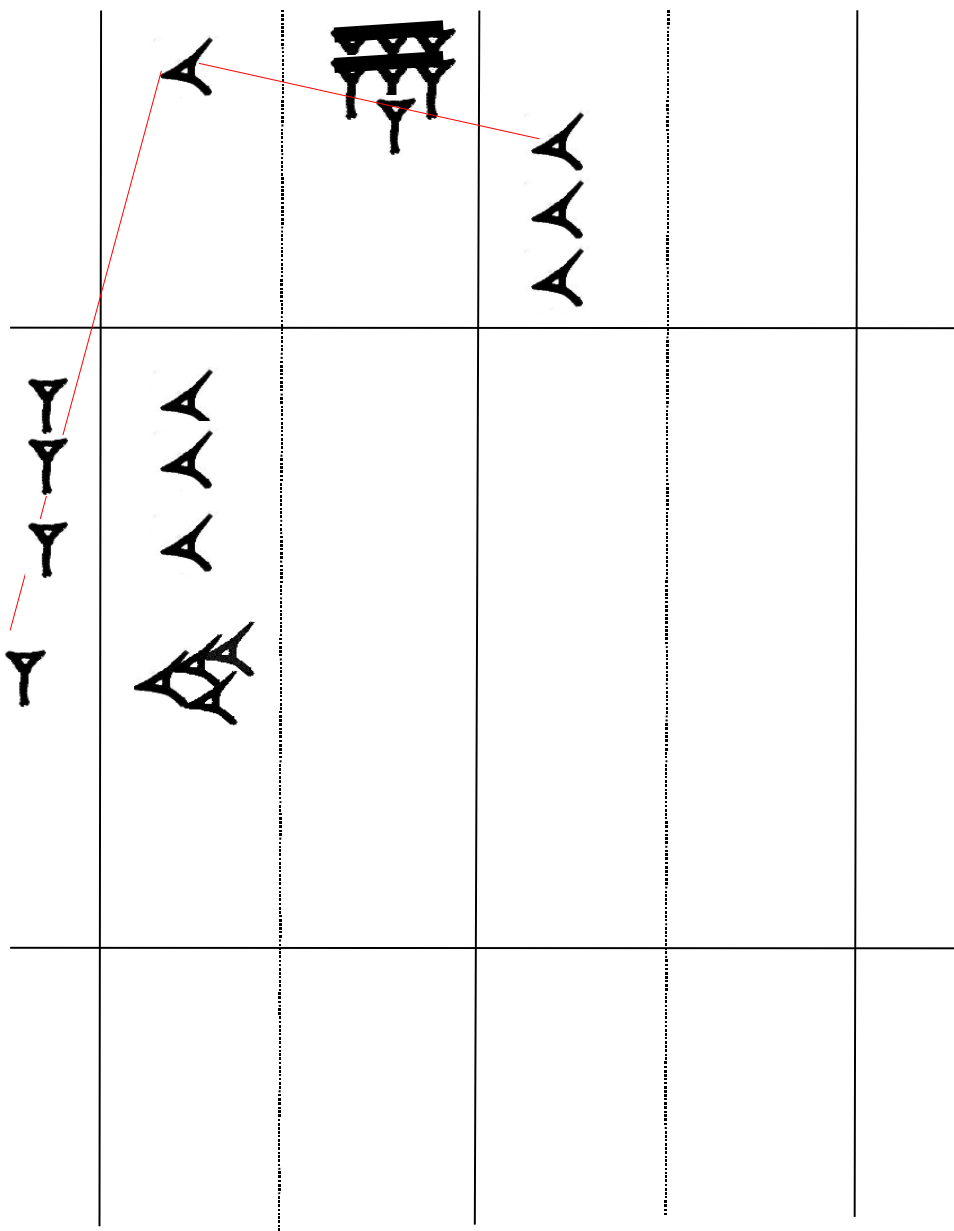
17 00 x 00 30 = ?



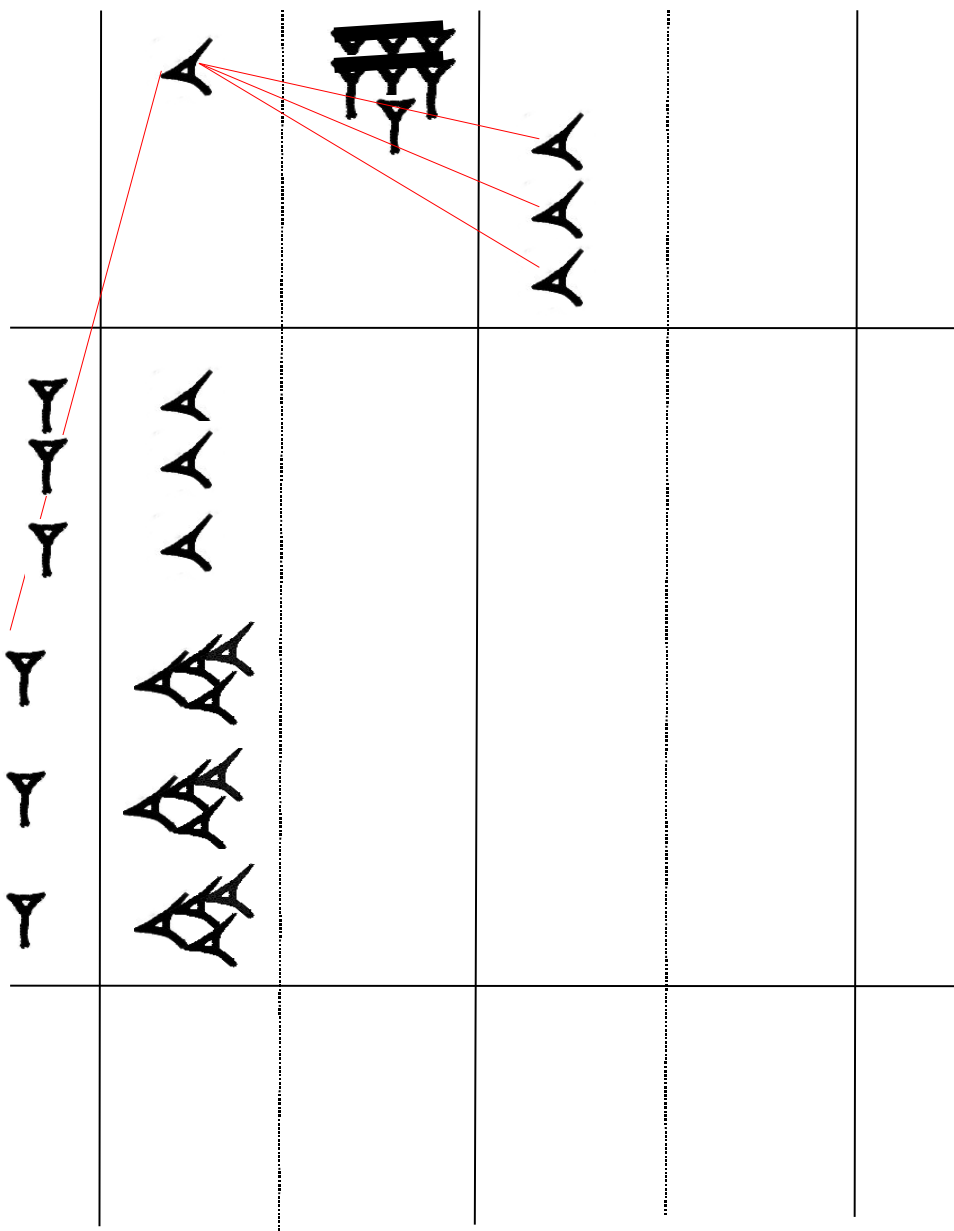
$A \times A = Y \text{ } \overline{AA}$

$10 \times 10 = 100$   
 $10 \times 10 = [1 \ 40]$

$$17\ 00 \times 00\ 30 = ?$$



$$17\ 00 \times 00\ 30 = ?$$














$$1700 \times 0030 = ?$$

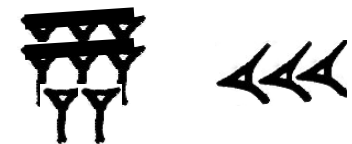
	A	<del>AAA</del> A		
Y Y Y	AA AA A			
Y Y Y	<del>AA</del> <del>AA</del> AA			
<del>AAA</del> YY	AA			

$$1700 \times 0030 = ?$$

	A	<del>AAA</del>	AAA	
Y Y Y	AA AA A			
Y Y Y	<del>AA</del> <del>AA</del> <del>AA</del>			
<del>AAA</del> YY	<del>AA</del>			

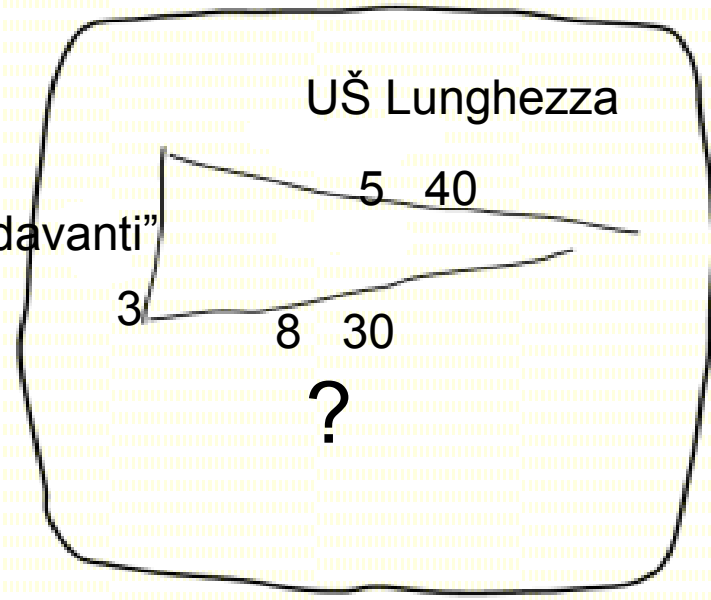
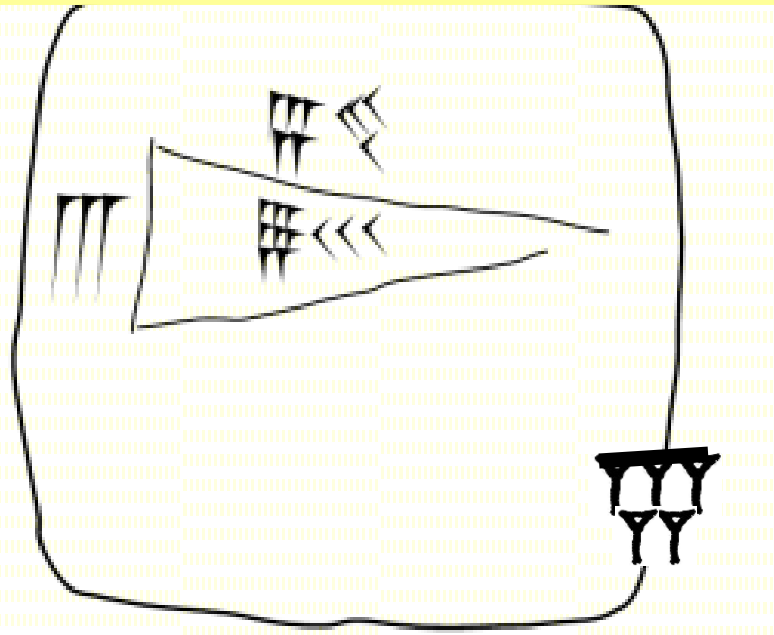
$$17\ 00 \times 00\ 30 = 8\ 30$$

	A		
			
			
			
			



L'area di un triangolo

SAG "davanti"



5 40 x 3 00 x 30



volte

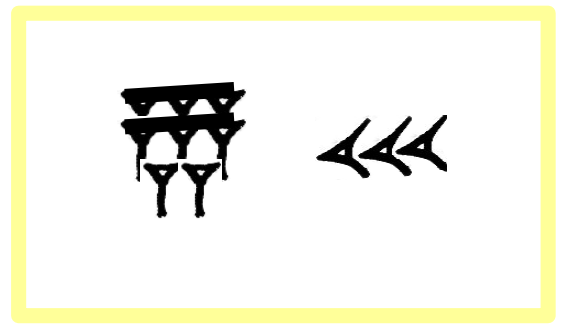


volte



A=?

$$A = (b \times h) / 2$$



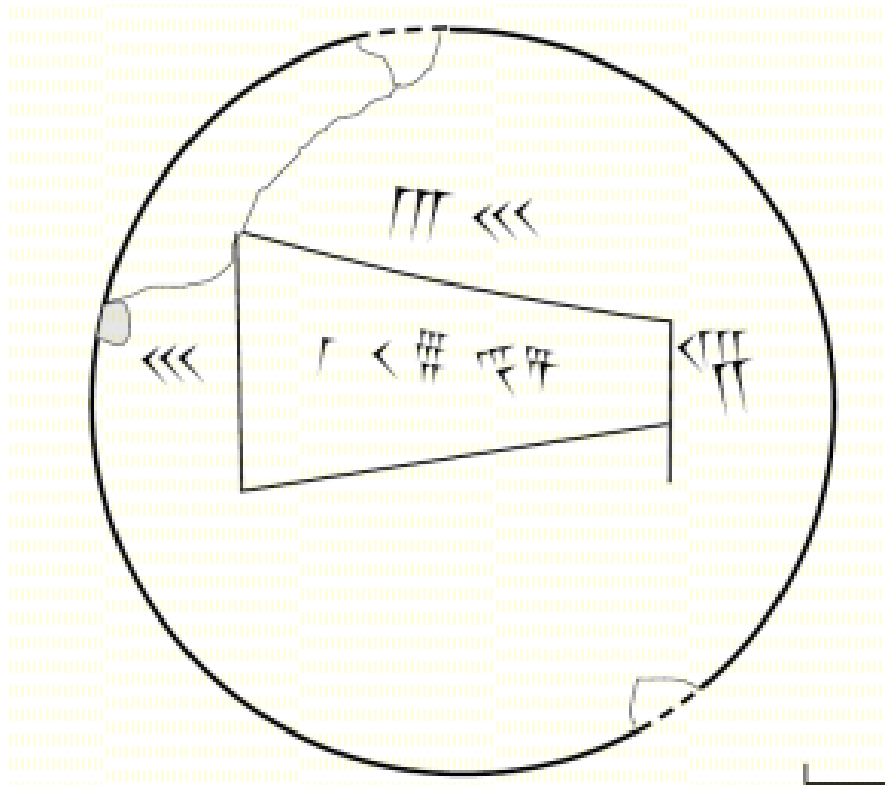
MS 2107 Schoyen collection

L'area di un trapezio



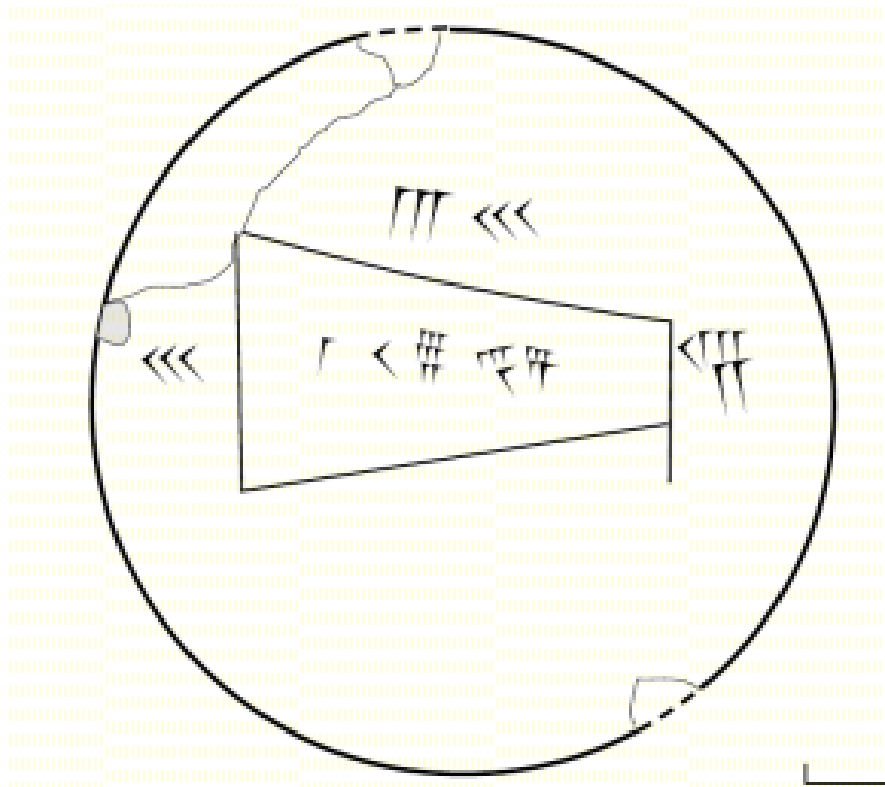
MS 2107 Schoyen collection

L'area di un trapezio



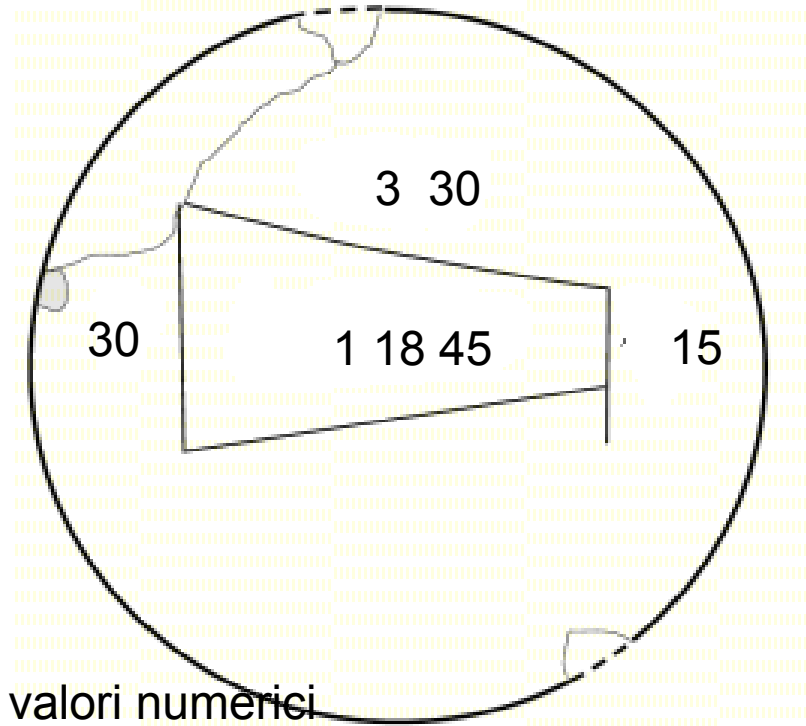
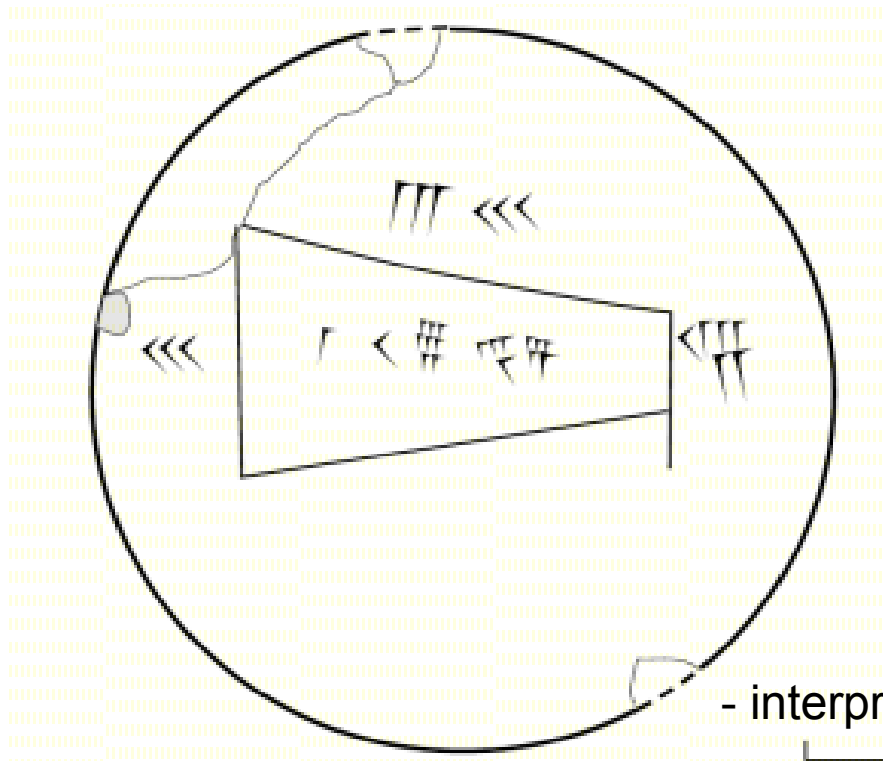
MS 2107 Schoyen collection

L'area di un trapezio



Osservazione: I trapezi sono molto comuni

L'area di un trapezio



- interpretazione valori numerici

3 30 -----> 3, 30 ninda (notazione sessagesimale)  
15 -----> 15 ninda  
30 -----> 30 ninda

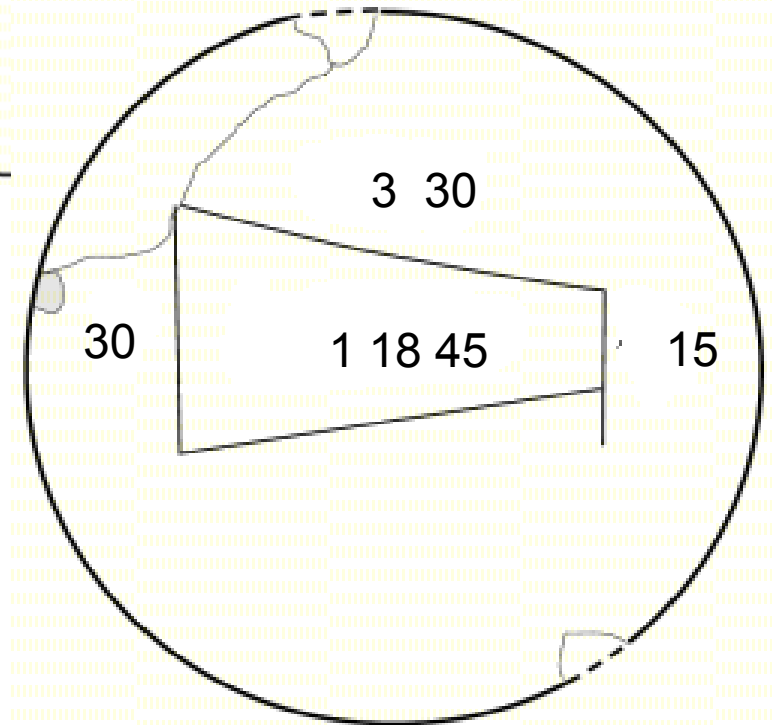
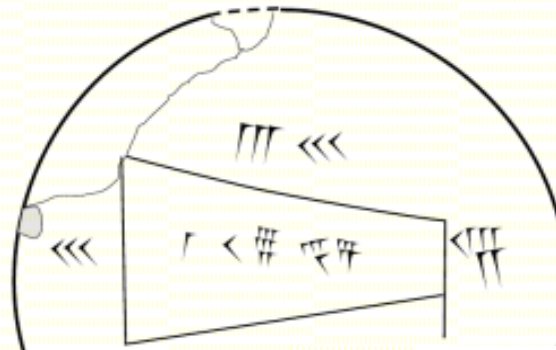
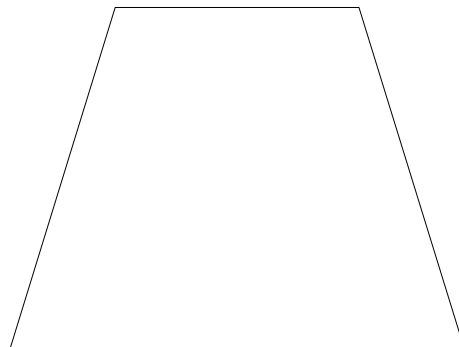


MS 2107 Schoyen collection

L'area di un trapezio

Dati: lunghezze  
Incognite: area

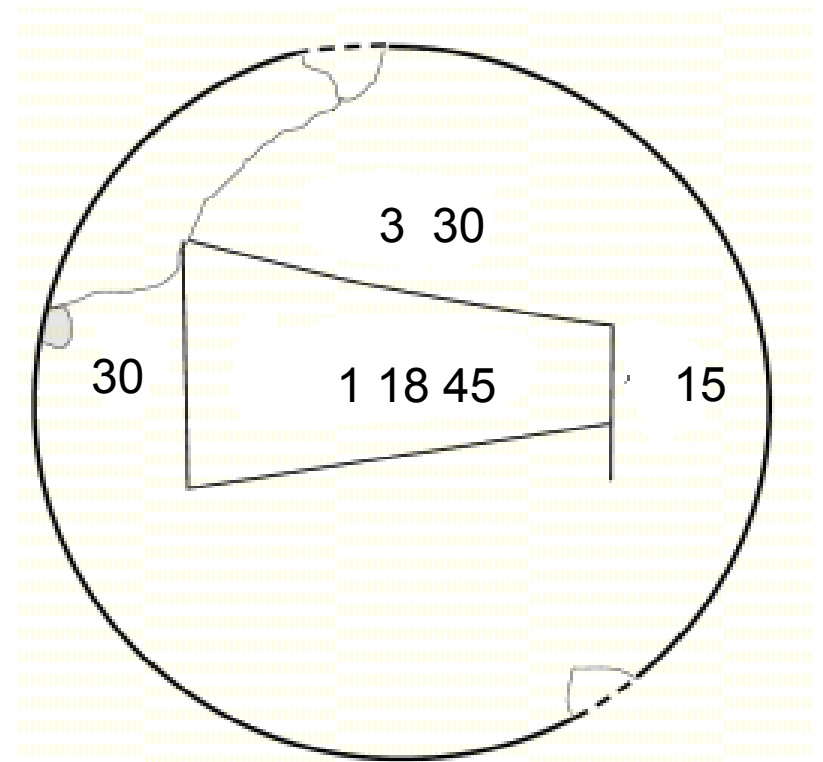
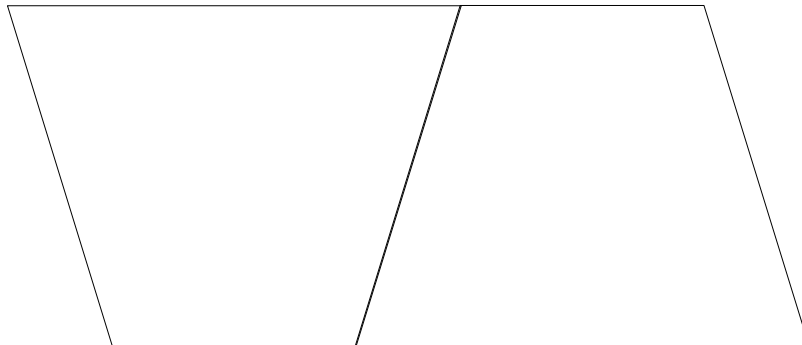
$$A = (B+b) \times h / 2$$



L'area di un trapezio

Dati: lunghezze  
Incognite: area

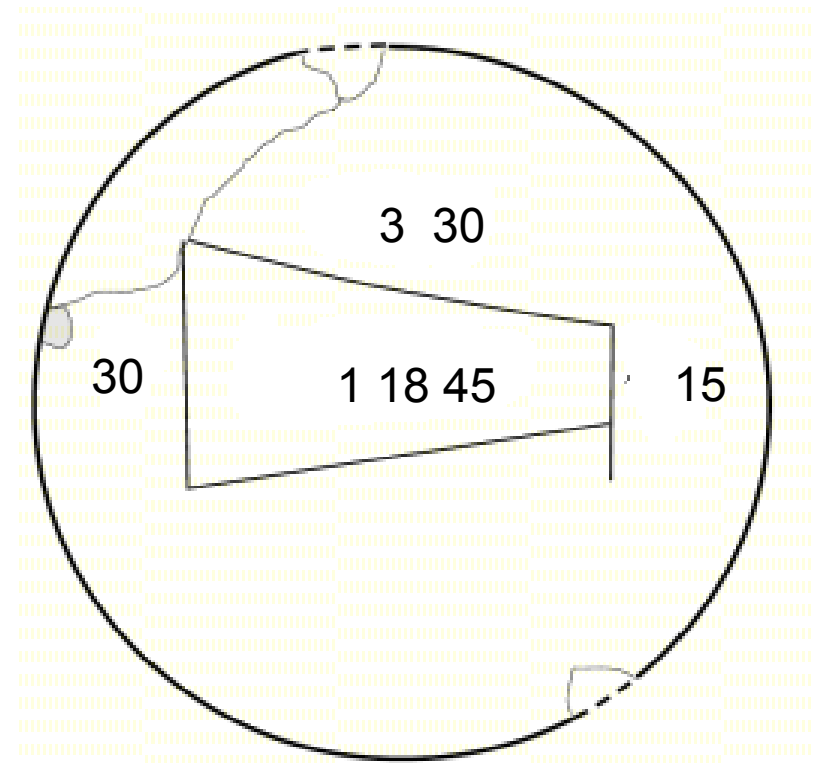
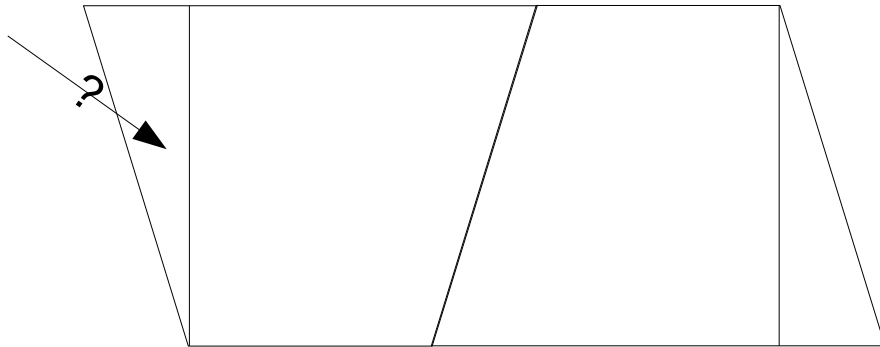
$$A = (B+b) \times h / 2$$



L'area di un trapezio

Dati: lunghezze  
Incognite: area

$$A = (B+b) \times h / 2$$



L'area di un trapezio

Dati: lunghezze  
Incognite: area

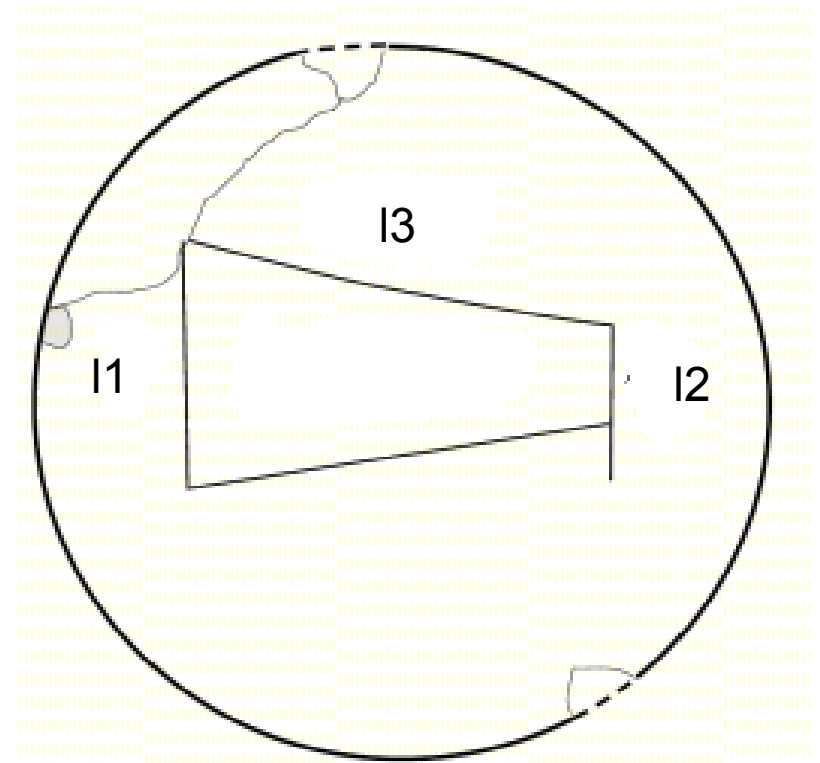
Formula del geometra

trapezi isosceli

$$A = (l_1 + l_2) \times l_3 \times 30$$

quadrilateri

$$A = (l_1 + l_2) \times (l_3 + l_4) \times 15$$



lati opposti

L'area di un trapezio

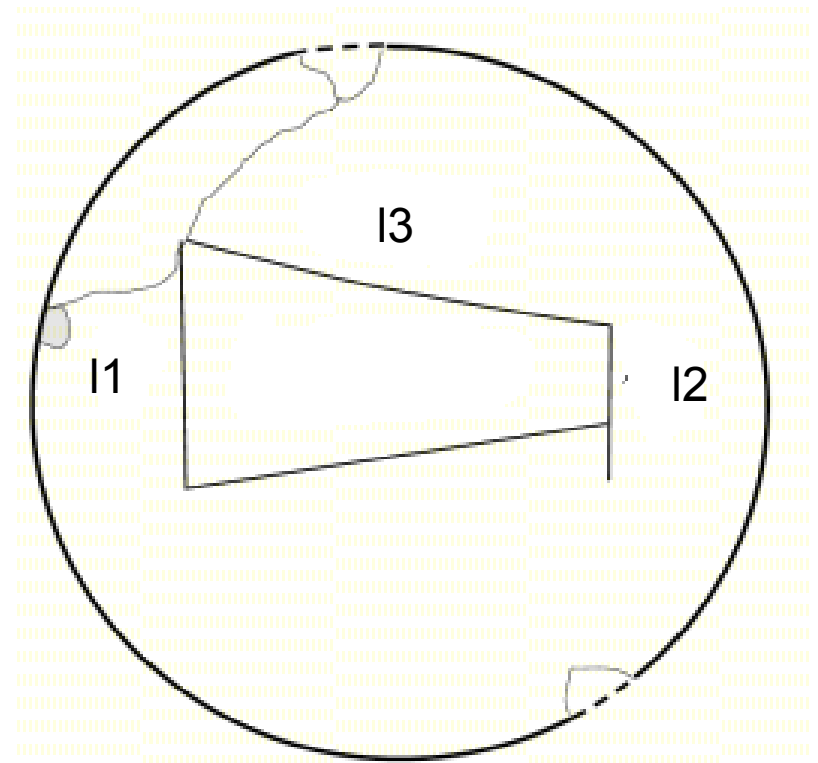
Dati: lunghezze  
Incognite: area

Formula del geometra

trapezi isosceli

$$A = (l_1 + l_2) \times l_3 \times 30$$

↑  
altezza  
lato obliquo



L'area di un trapezio

Dati: lunghezze  
Incognite: area

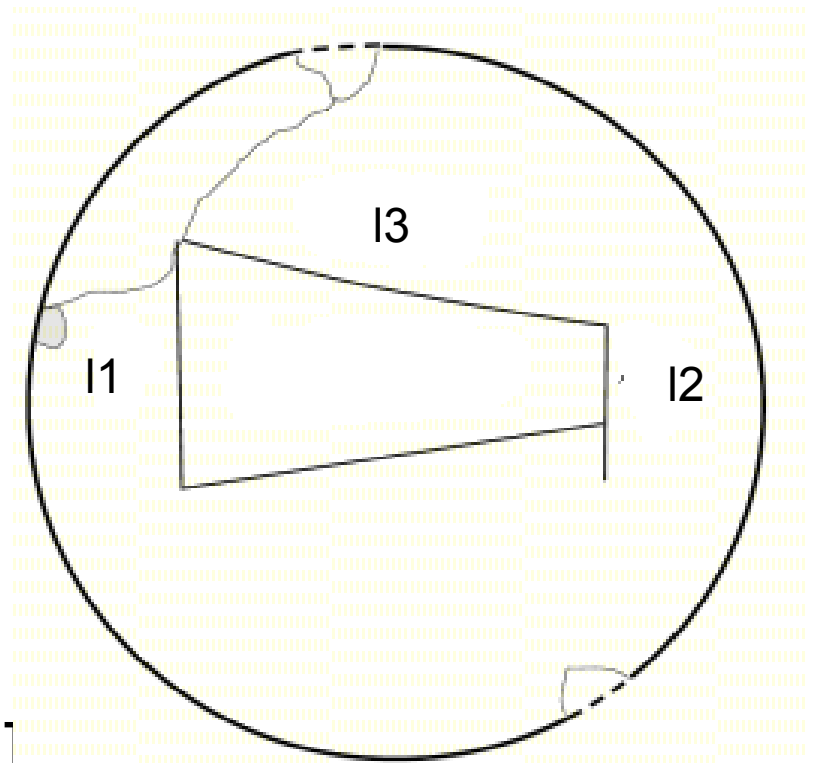
Formula del geometra

trapezi isosceli

$$A = (l_1 + l_2) \times l_3 \times 30$$

$$A = (l_1 + l_2) \times l_3 / 2$$

$$\begin{aligned} 30 &= [00; 30] \\ &= 30/60 = 1/2 \end{aligned}$$



L'area di un trapezio

Dati: lunghezze  
Incognite: area

Formula del geometra

trapezi isosceli

$$A = (l_1 + l_2) \times l_3 \times 30$$

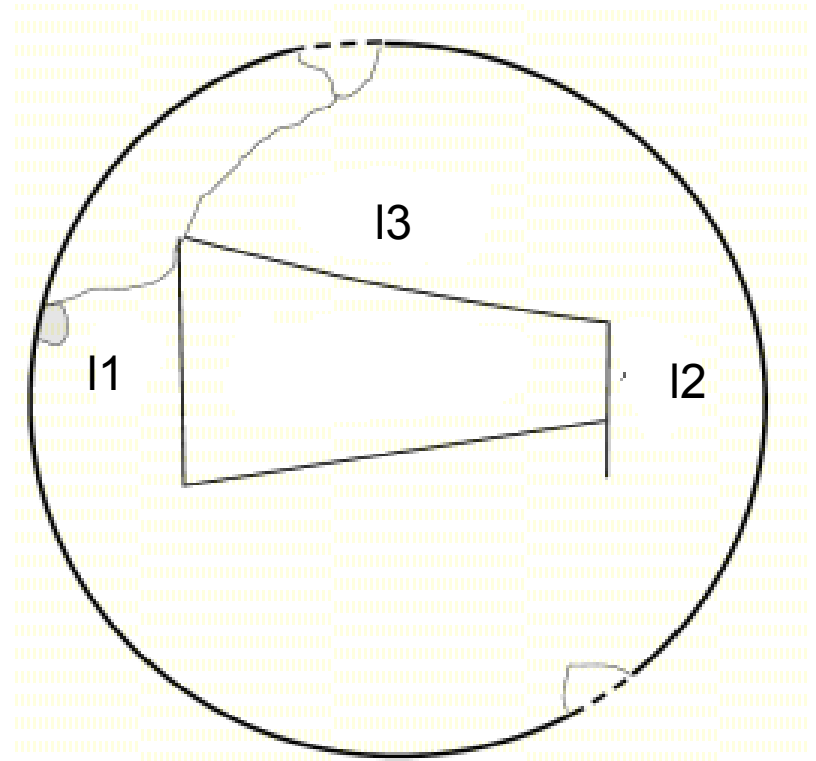
quadrilateri

$$15 = [00; 15] = 1/4$$

$$A = (l_1 + l_2) \times (l_3 + l_4) \times 15$$

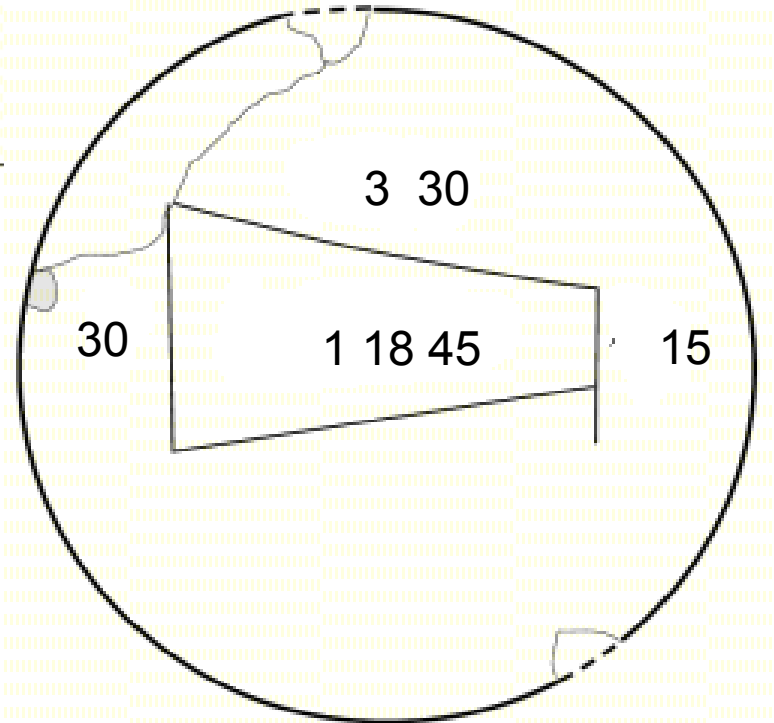
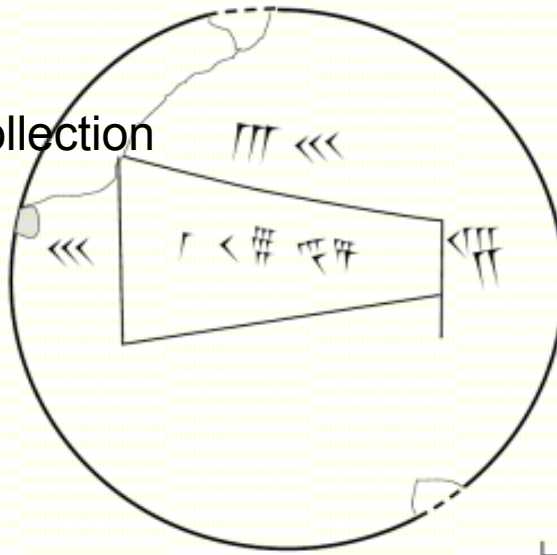
$$A = (l_1 + l_2) \times (l_3 + l_4)$$

$$\frac{\quad}{2} \quad \frac{\quad}{2}$$



MS 2107 Schoyen collection

L'area di un trapezio



Dati: lunghezze  
Incognite: area

$$A = (I1+I2) \times h \div 2$$

Calcoliamo

$$(30+15) \times 30 \div 2$$

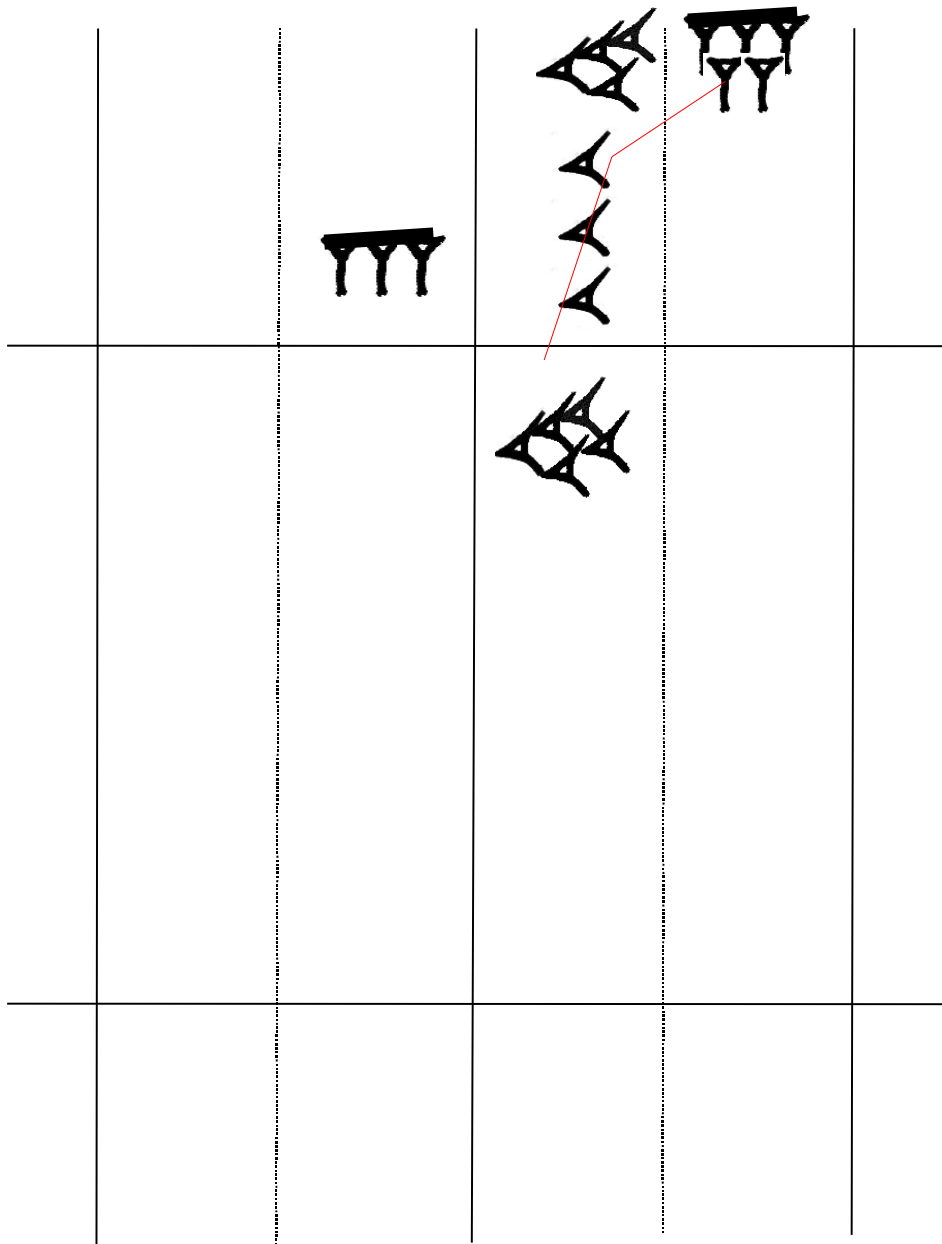
30+1  
5





$(30+15) \times 3$   
 $30 \times$   
 $30$

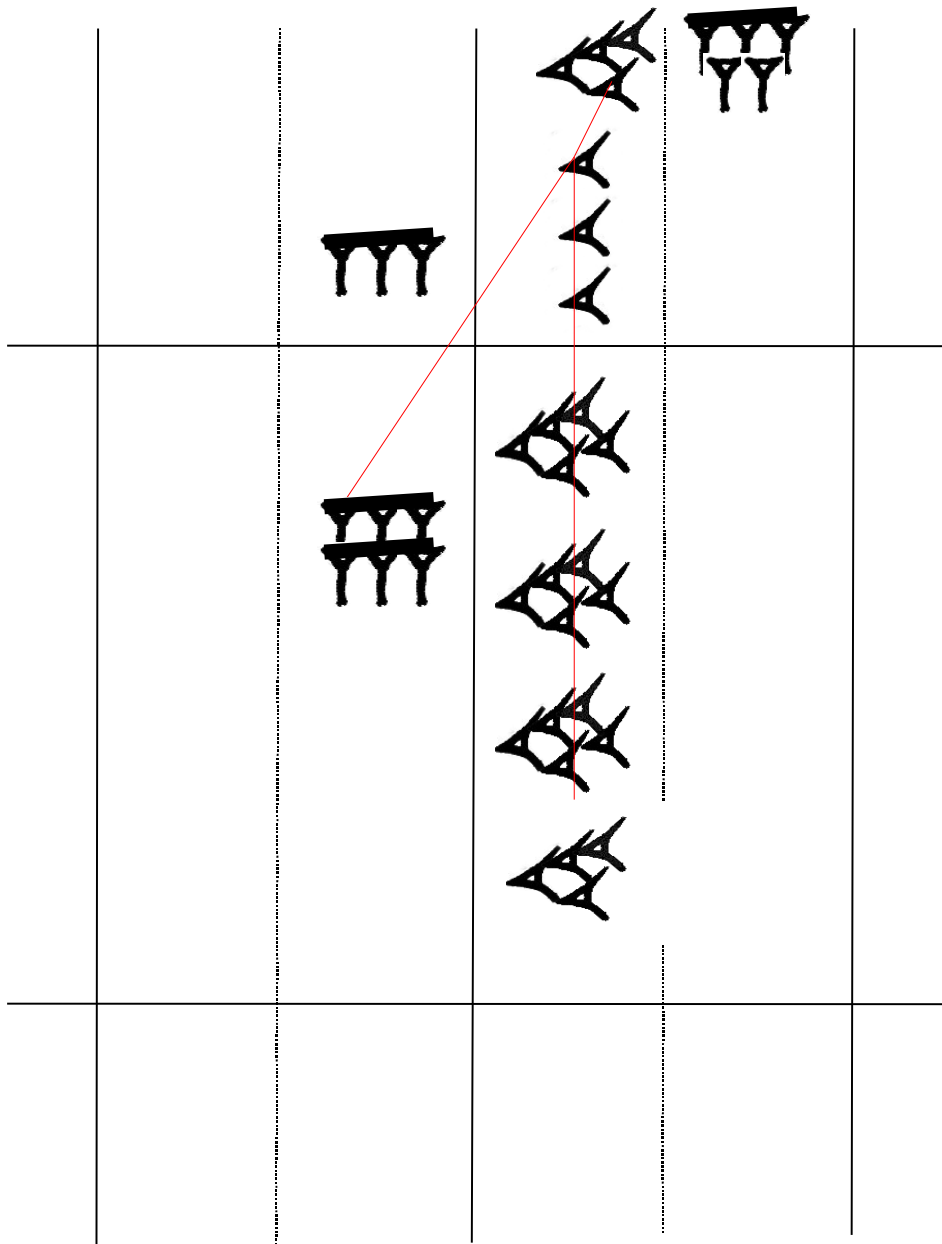
$45 \times 3 = ?$

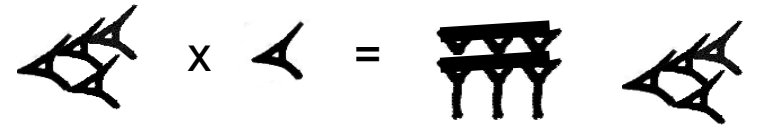


$\text{Fish with 3 fins} \times \text{Fish} = \text{Fish with 3 fins}$



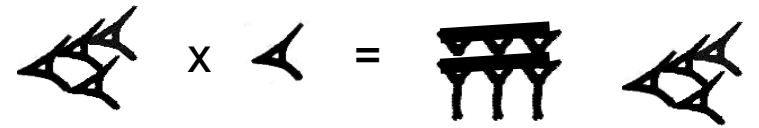
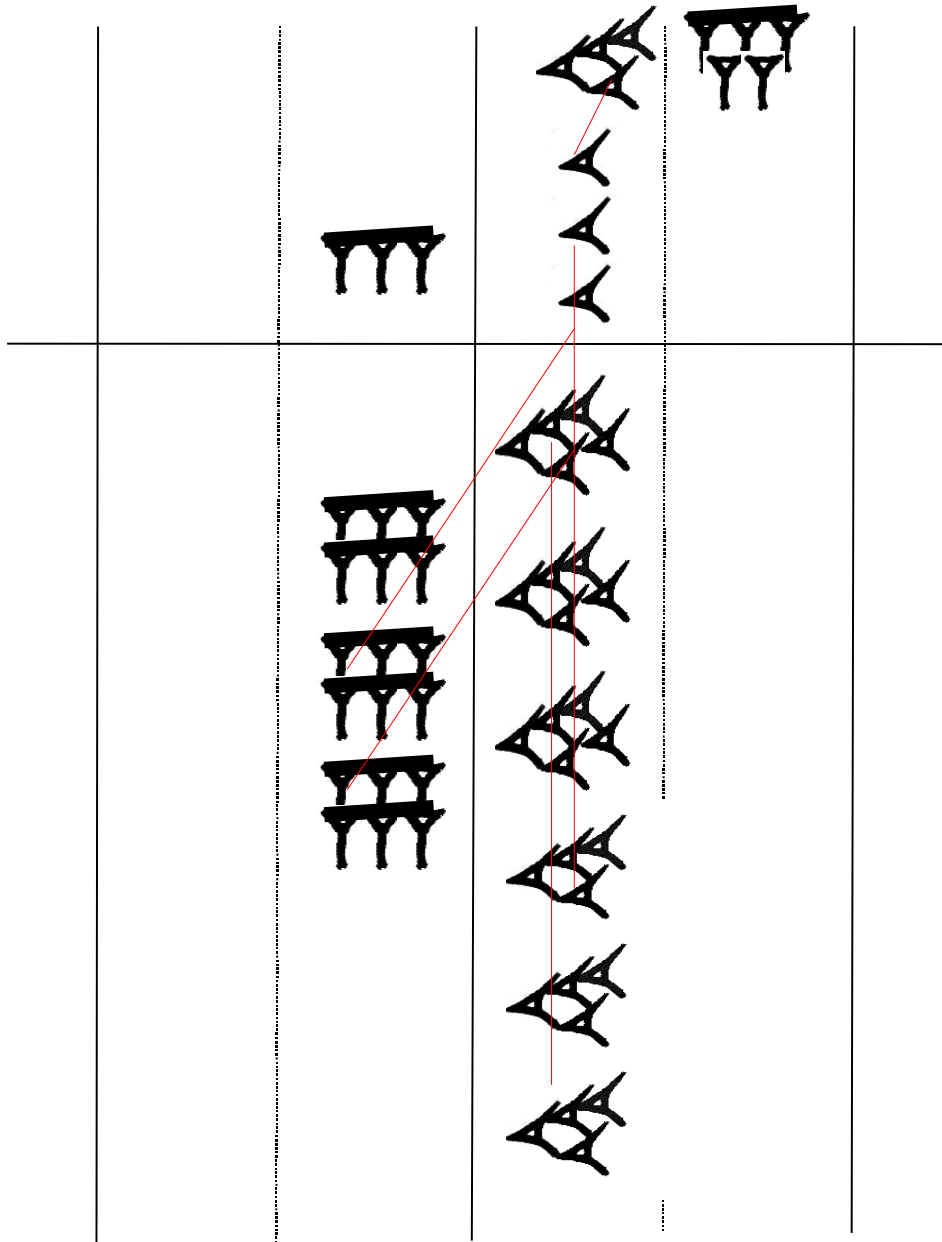
45 x 330 = ?





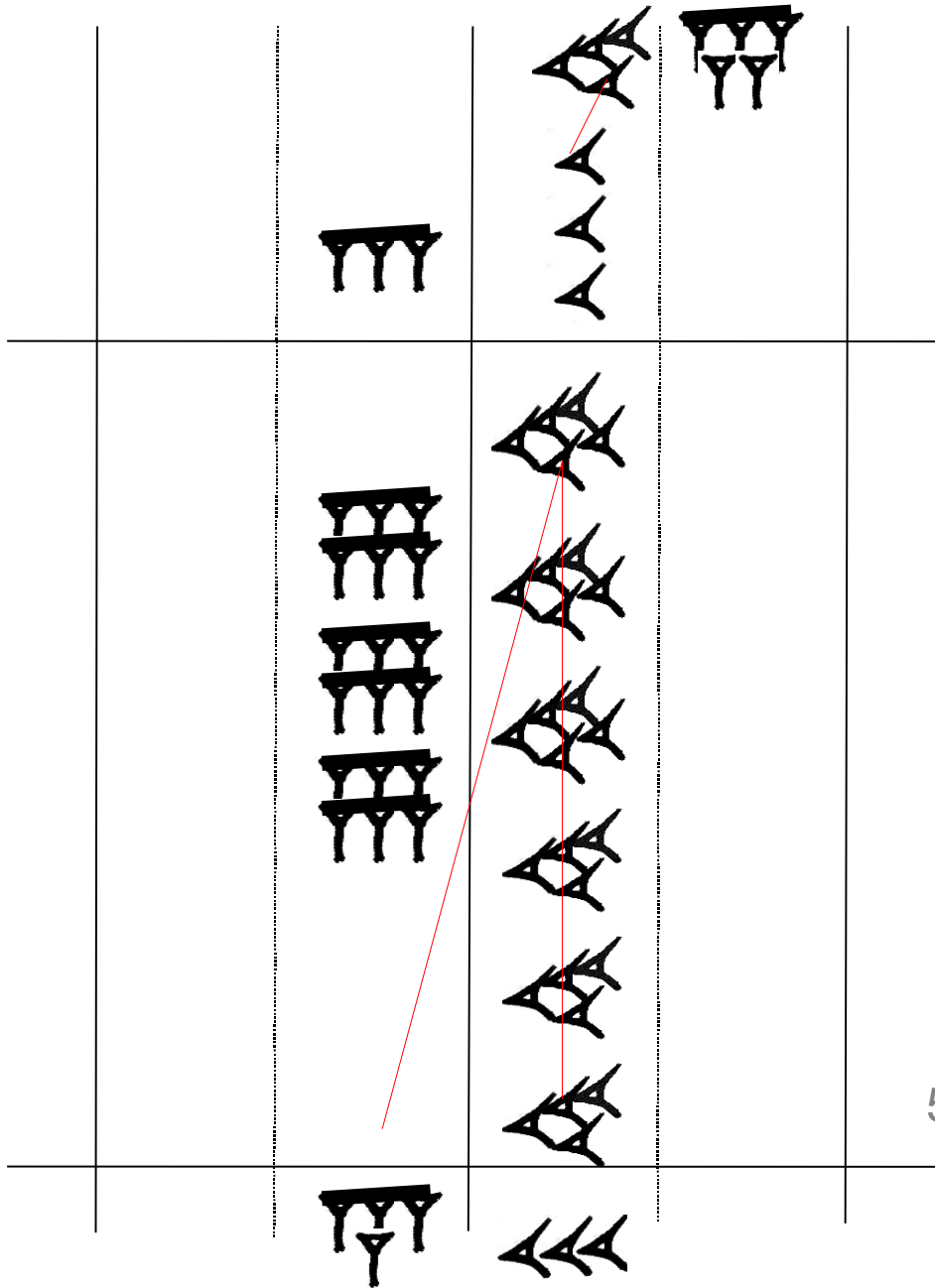
40x10=400  
40x10=[6 40]

45 x 330 = ?



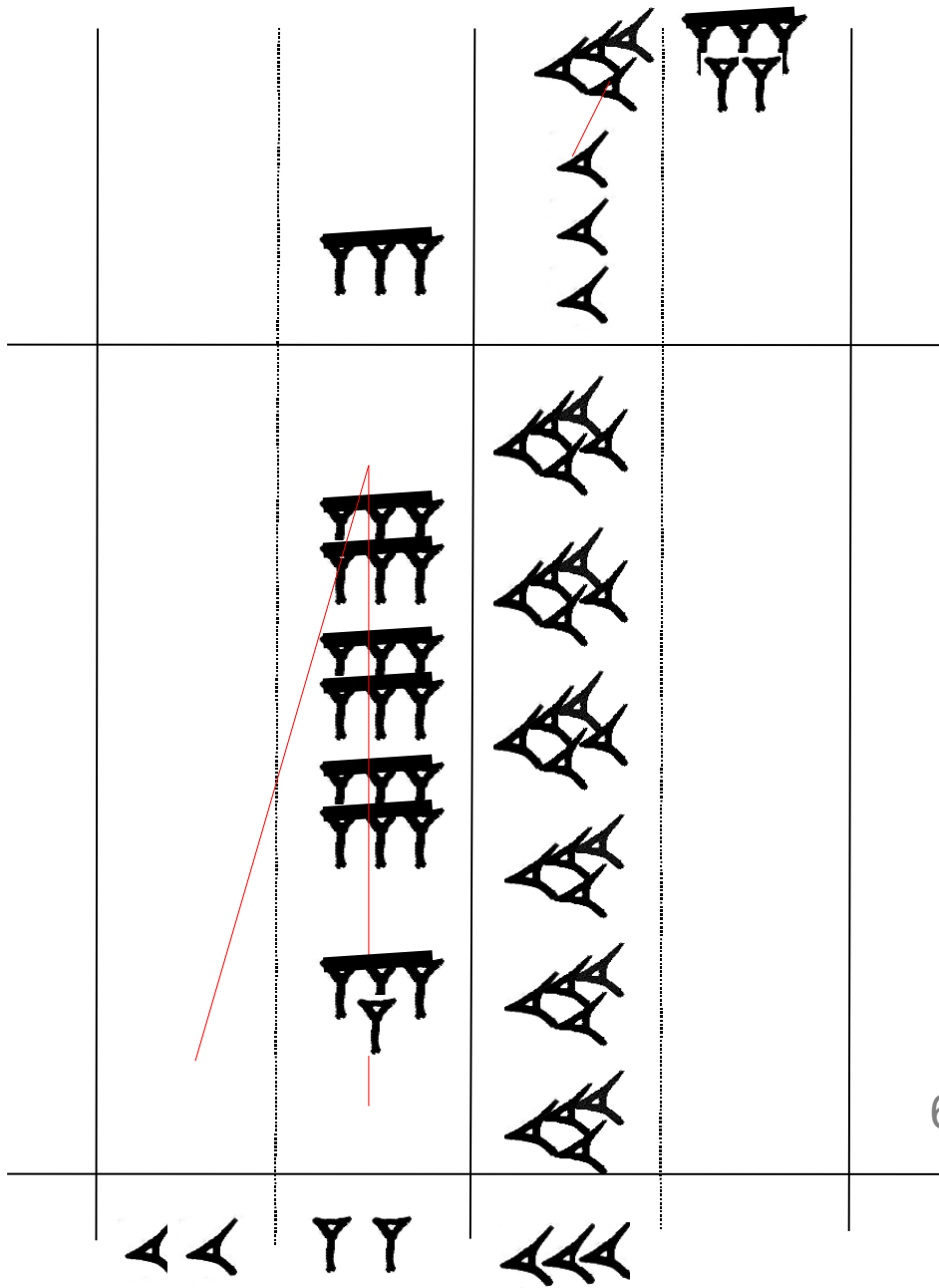
40x10=400  
40x10=640

$45 \times 330 = ?$



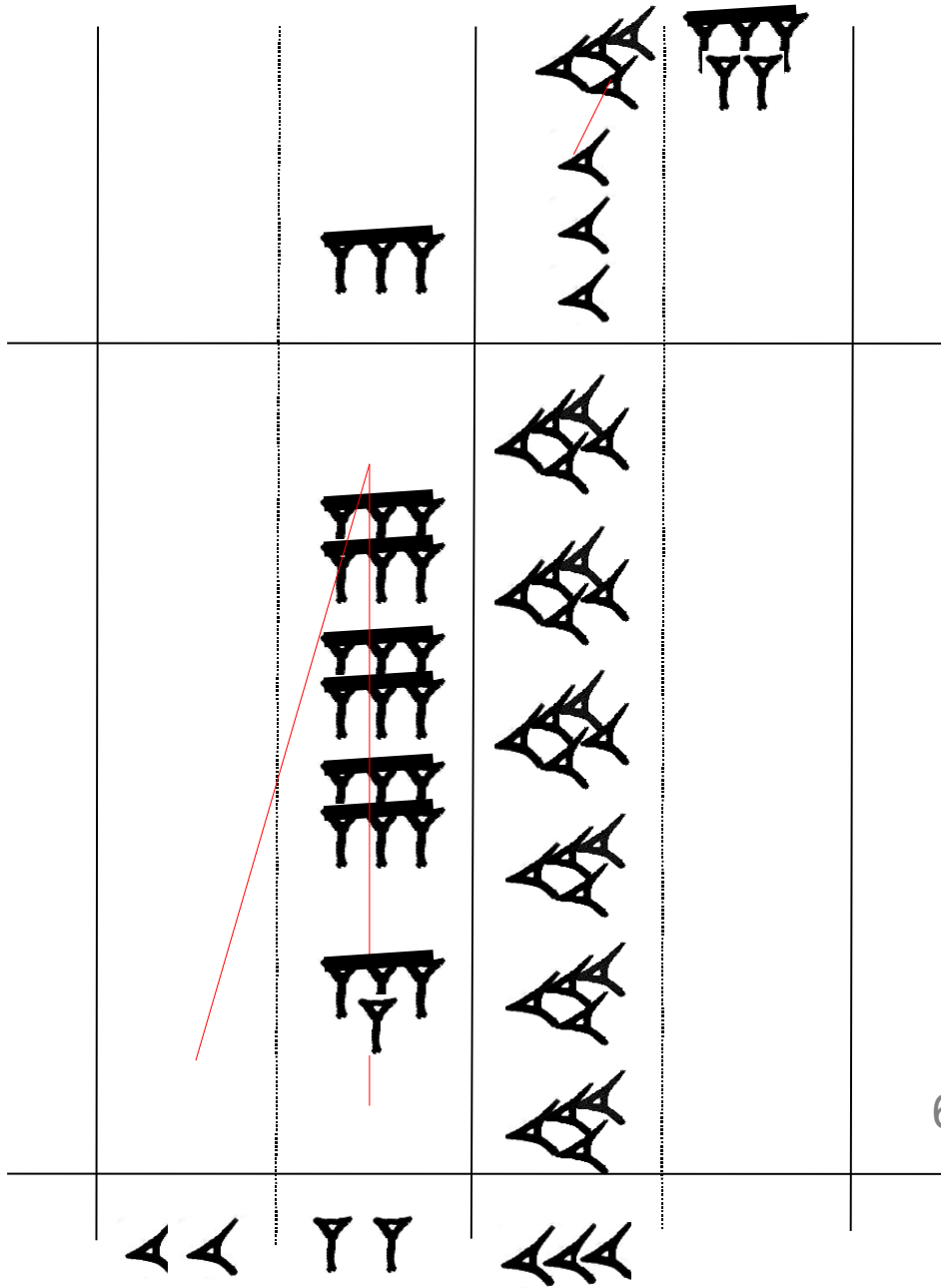
$50+50+50+40+40+40=270=4 \times 60+30$

$45 \times 330 = ?$



$6+6+6+4=22$

$$45 \times 30 = 22 \ 30$$



$$6+6+6+4=22$$

$45 \times 30 = 2230$



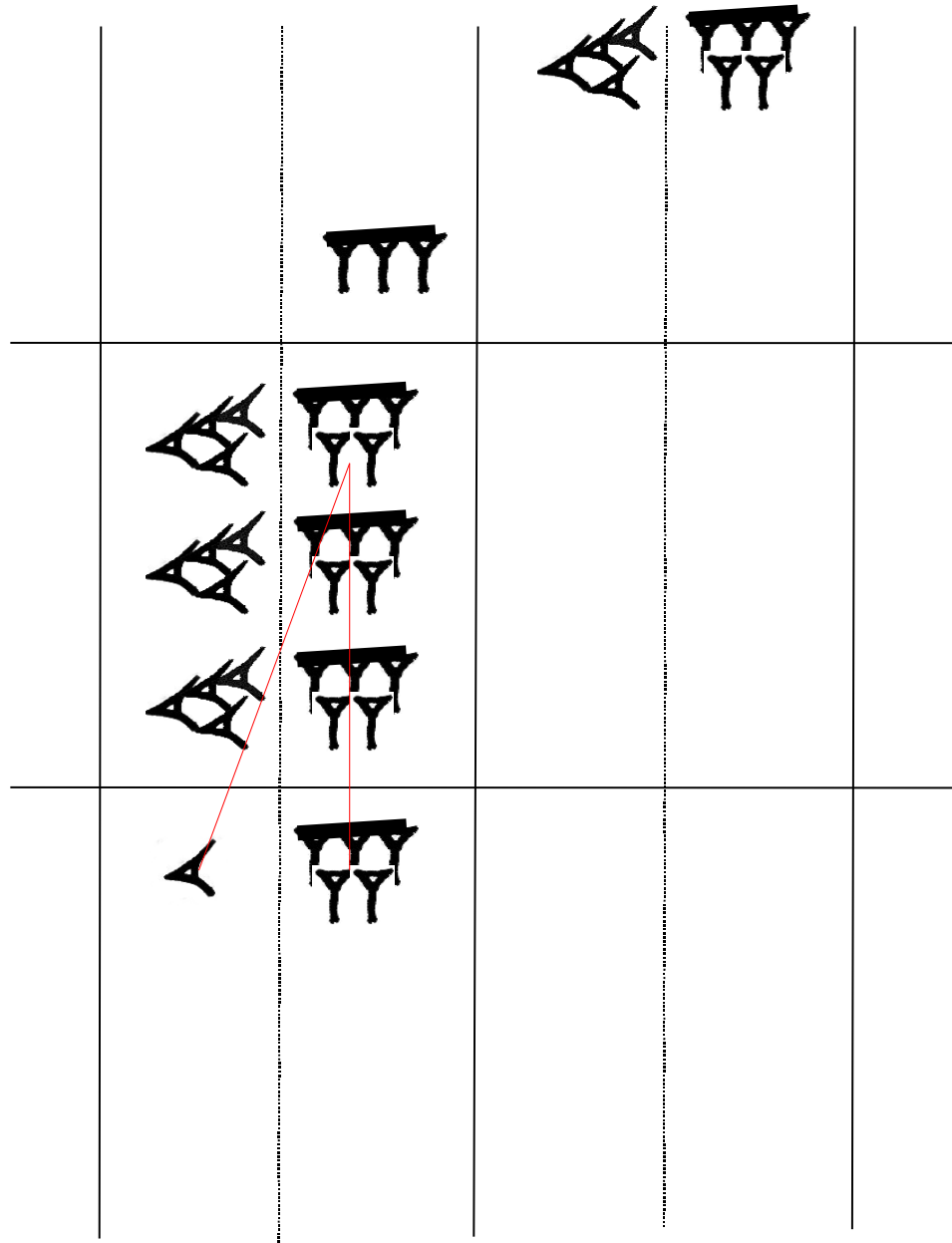
$45 \times 3 = ?$




$45 \times 30 = 2230$



$45 \times 3 = ?$



$45 \times 30 = 2230$

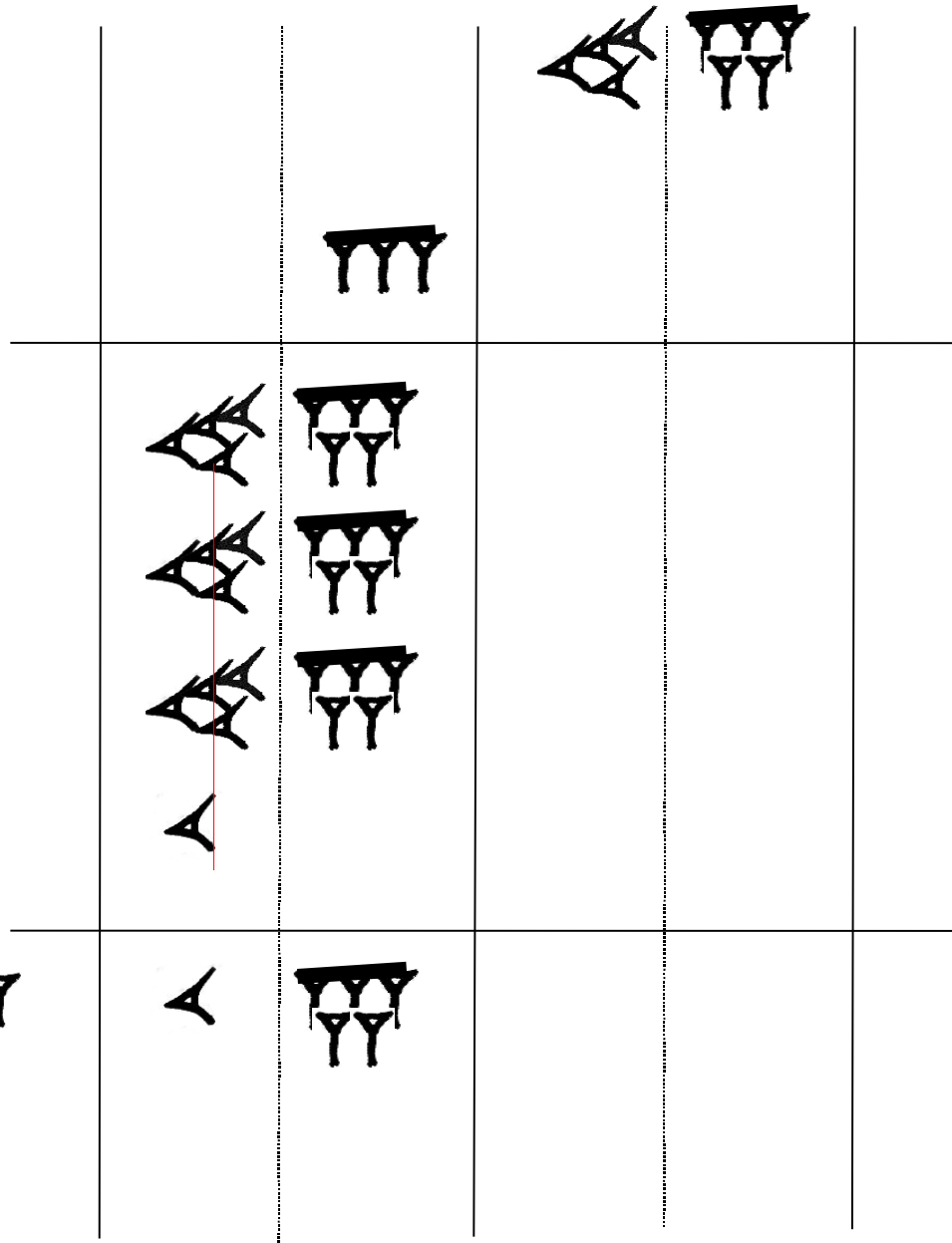


$45 \times 3 = ?$


$45 \times 30 = 2230$



$45 \times 3 = ?$

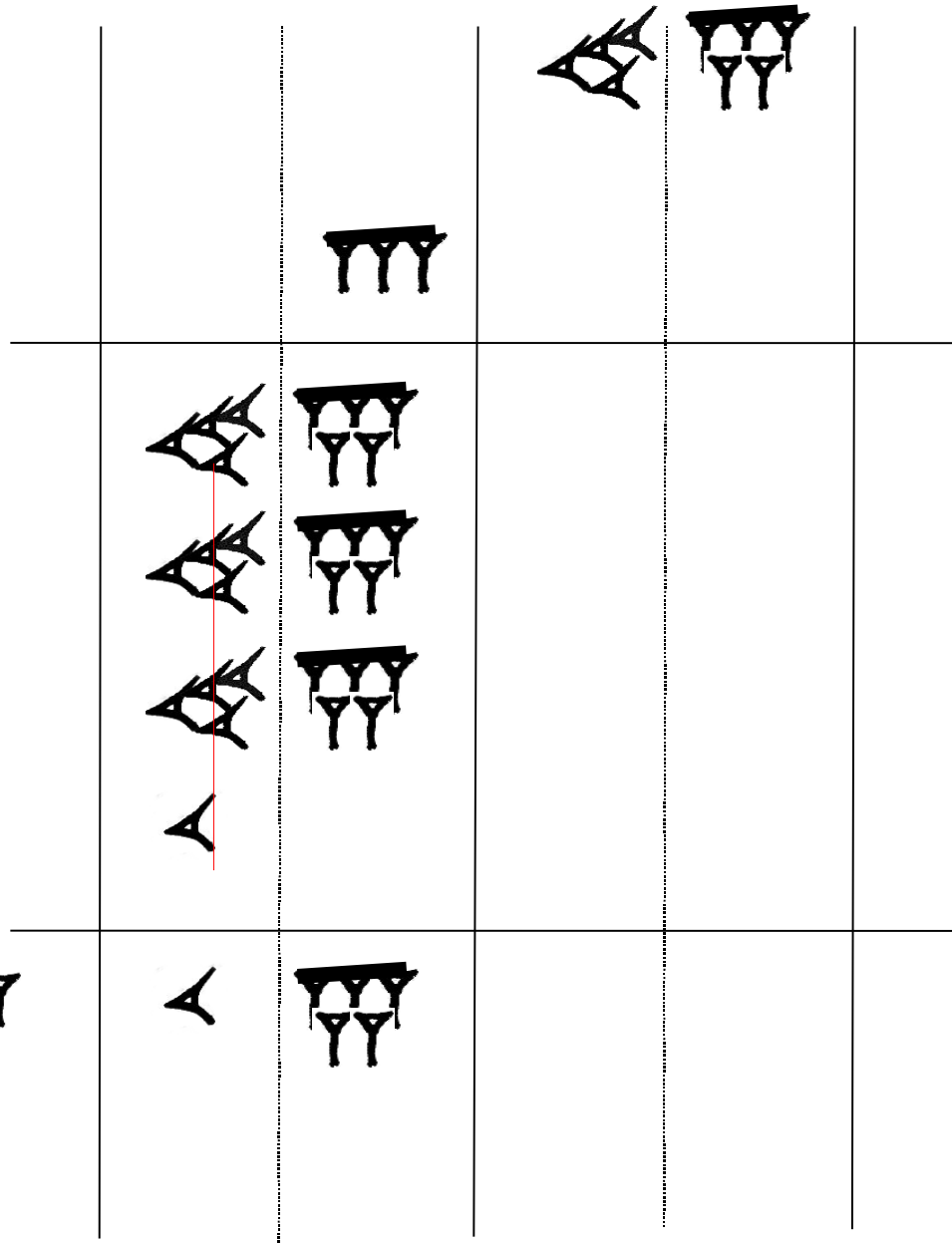


$4+4+4+1=13$  decine =  
2 sessantine + 1 decina

$45 \times 30 = 2230$



$45 \times 3 = ?$










4+4+4+1=13 decine =  
2 sessantine + 1 decina

$45 \times 30 = 22\ 30$



$45 \times 3 = 2\ 15\ 00$

$45 \times 30 = 22\ 30$



$45 \times 3 = 2\ 15\ 00$

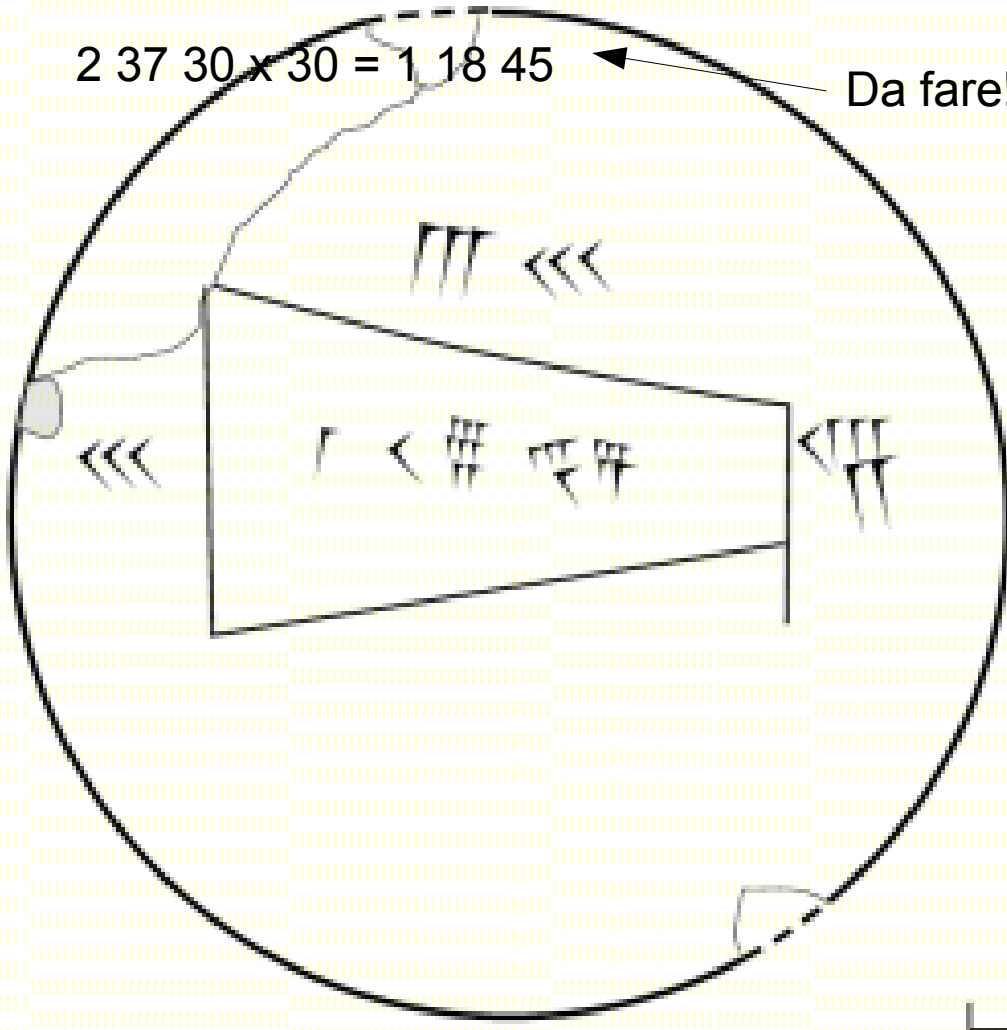
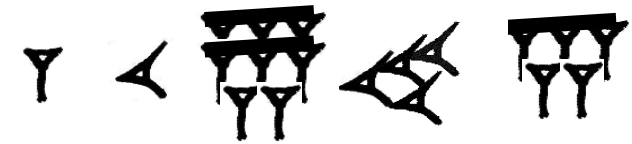


$45 \times 3\ 30 = 2\ 37\ 30$

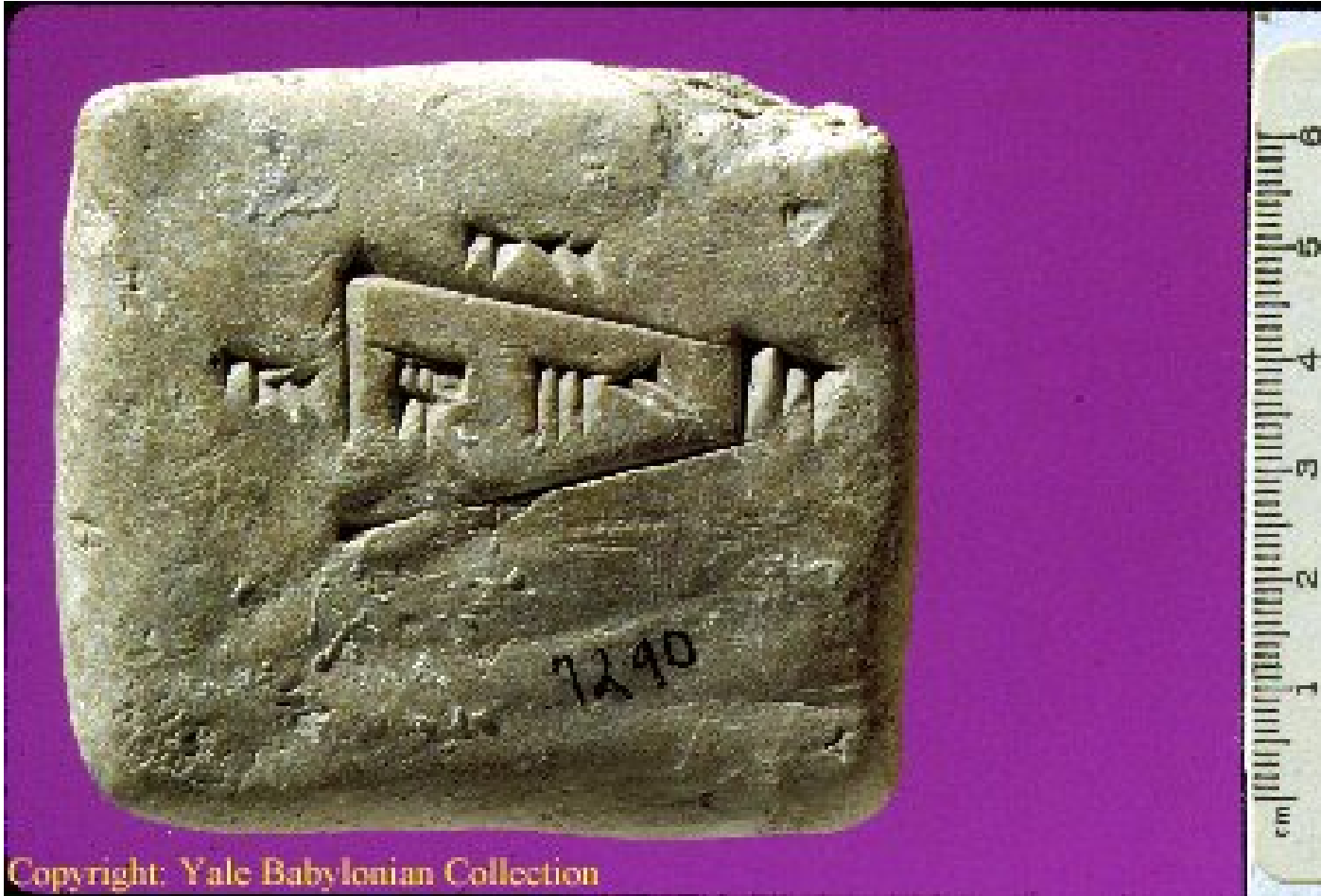


$2\ 37\ 30 \times 30 = 1\ 18\ 45$

Da fare!



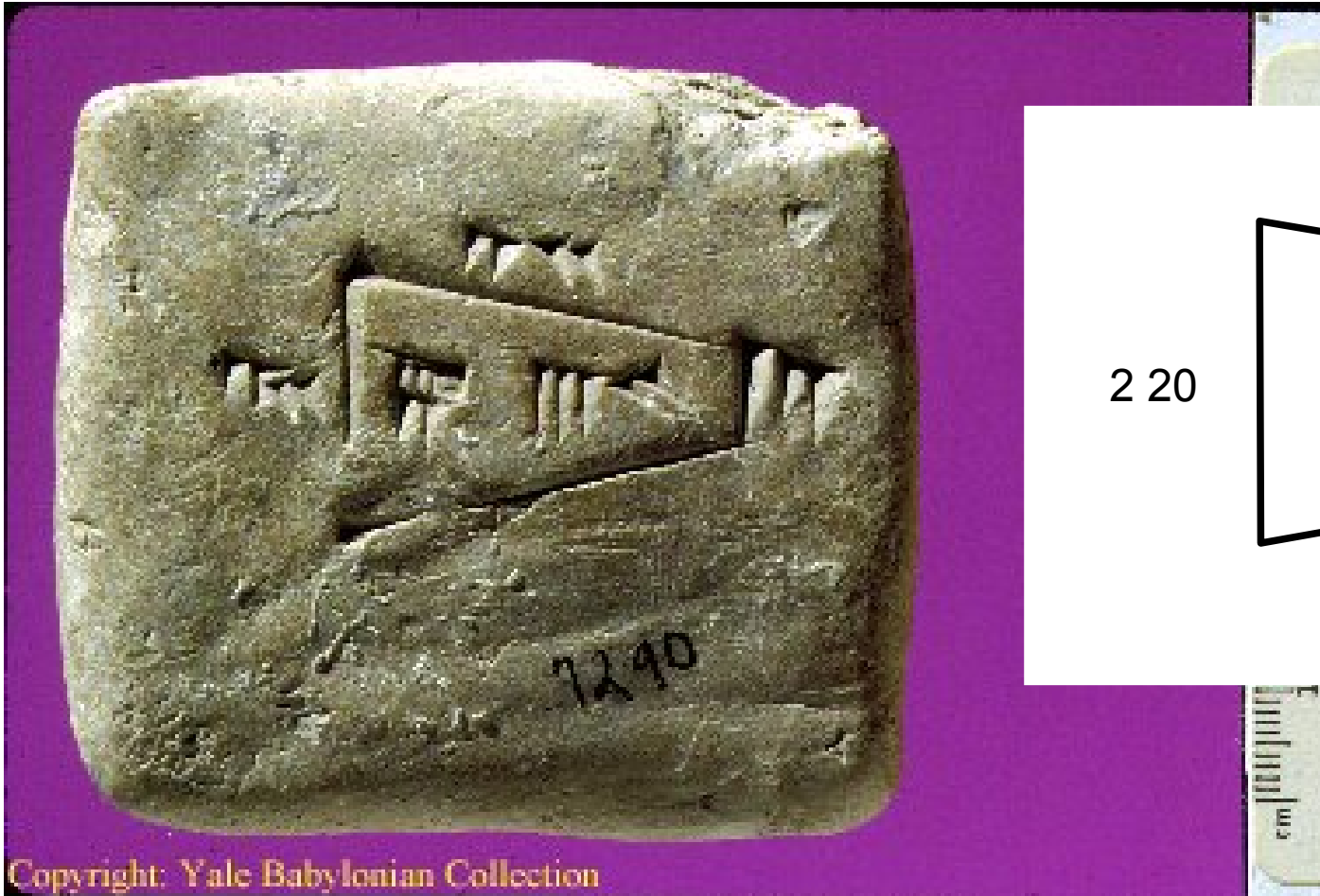
YBC 7240  
L'area di un trapezio (2)



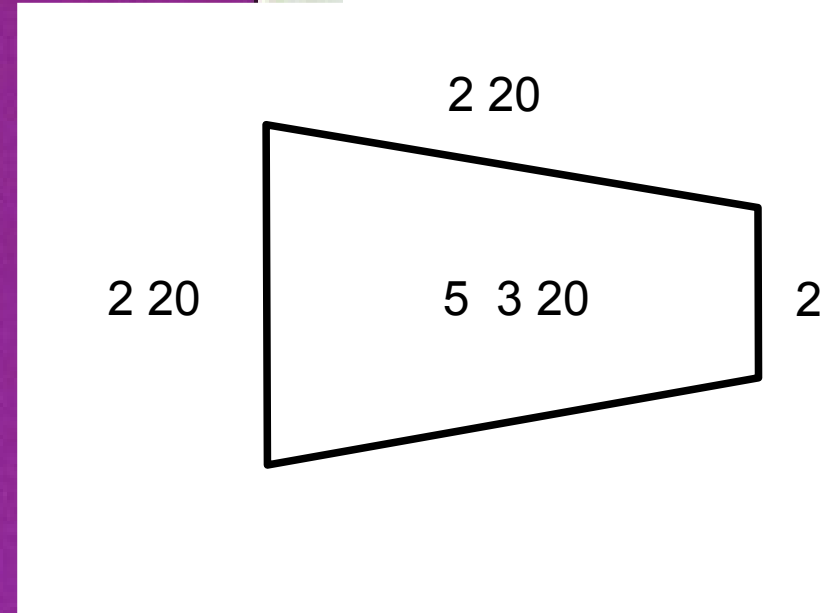
Copyright: Yale Babylonian Collection

YBC 7240

L'area di un trapezio (2)



Copyright: Yale Babylonian Collection



Formula del  
geometra  
trapezi isosceli  
 $A = (l_1 + l_2) \times l_3 \times 30$

$$5,3,20 = (2,20+2) \times 2,20 \times 30.$$



# CIRCONFERENZA e CERCHIO

Le “nostre” formule

circonferenza  
diametro

$$c=2\pi r$$

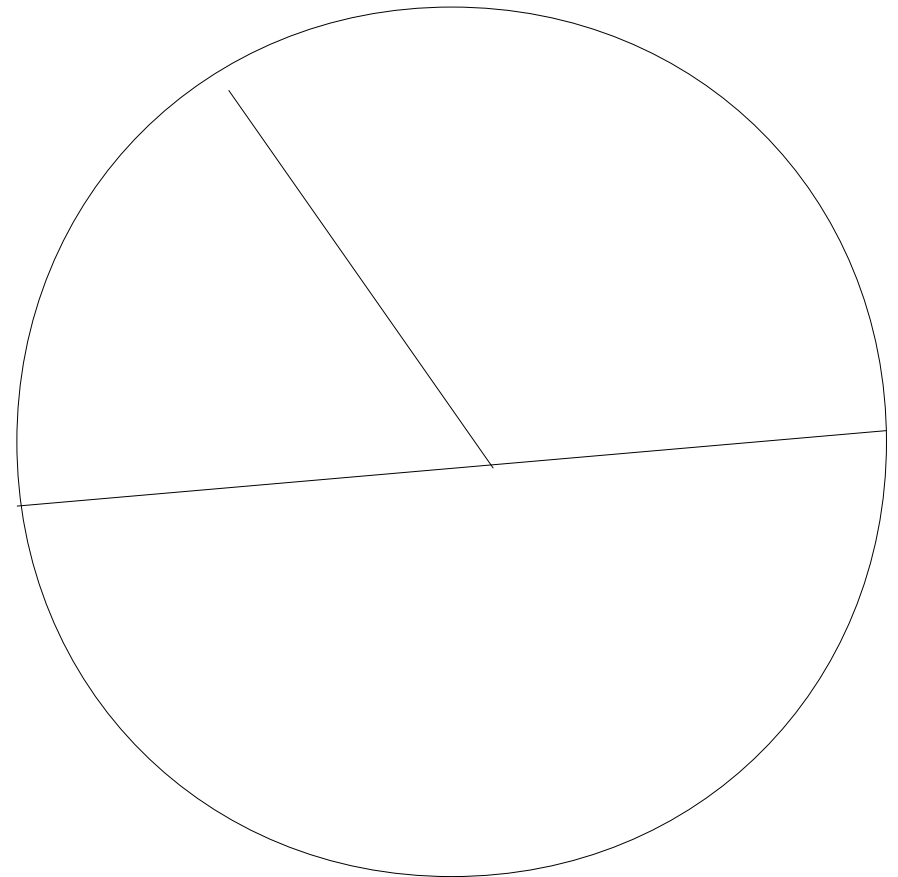
$$d=2r$$

$$c=\pi d$$

area

$$A= \pi r^2$$

Raggio (o diametro)  
misure fondamentali

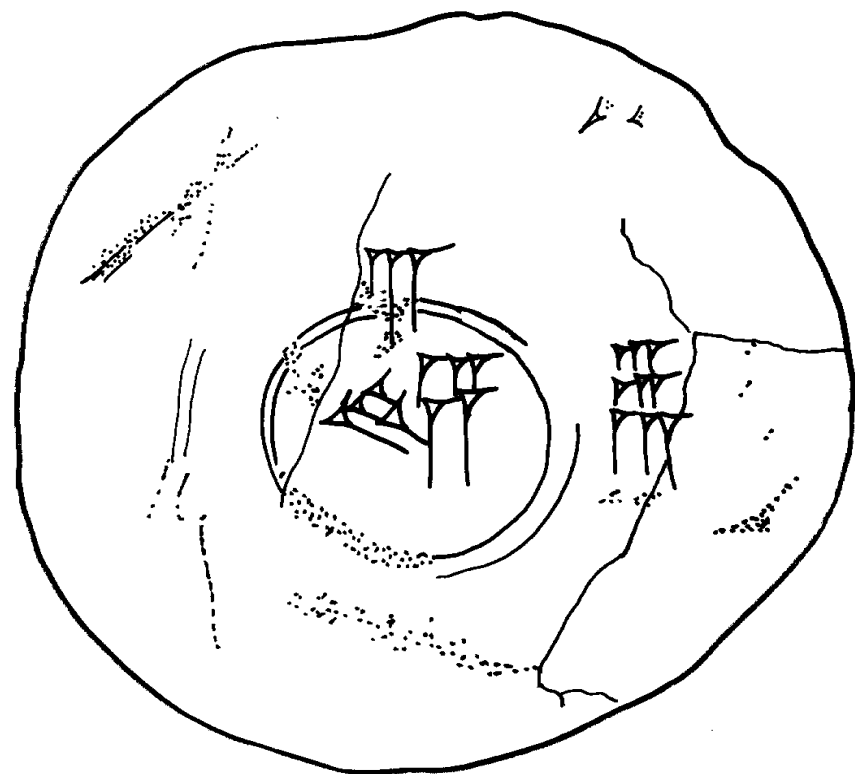


YBC 7302

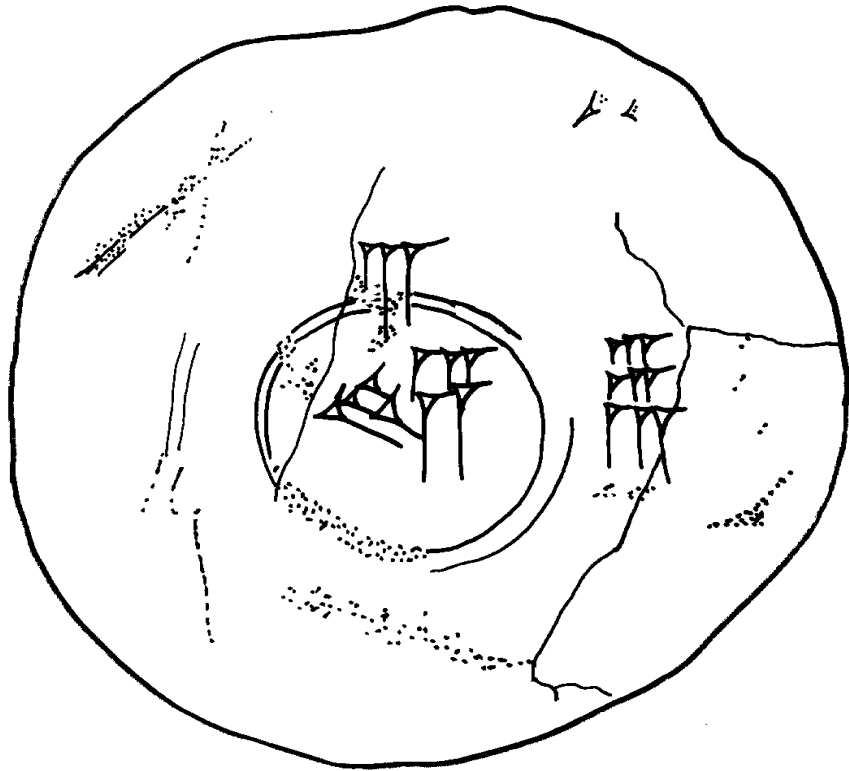


Copyright: Yale Babylonian

Che numeri?  
Che relazione fra loro?



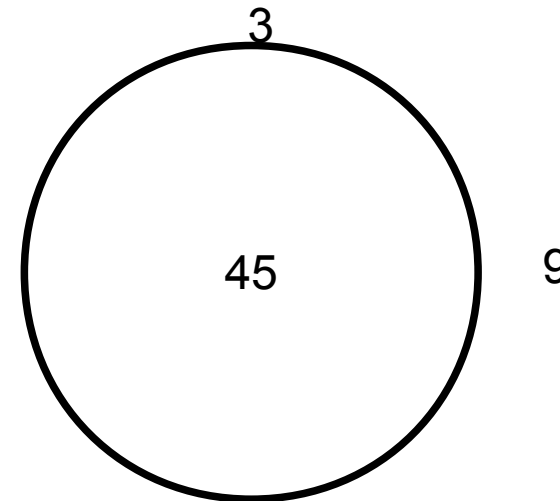
# YBC 7302



La loro posizione indica il significato:

3 si riferisce al contorno  
45 all'"interno"

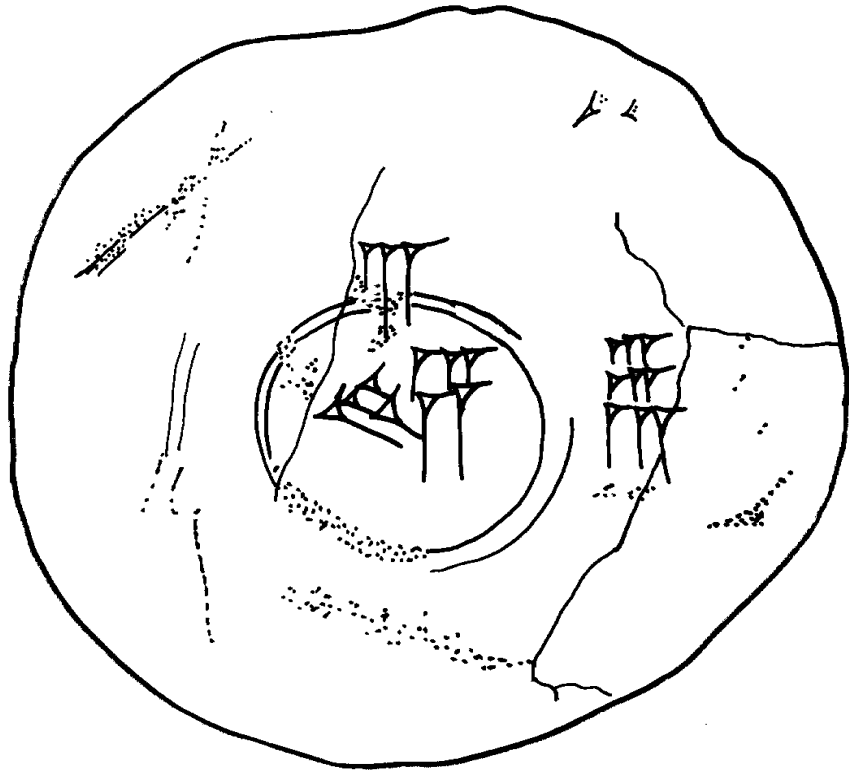
Che numeri?  
Che relazione fra loro?



$$9 = 3 \times 3$$

$$45 = 9 \times 5$$

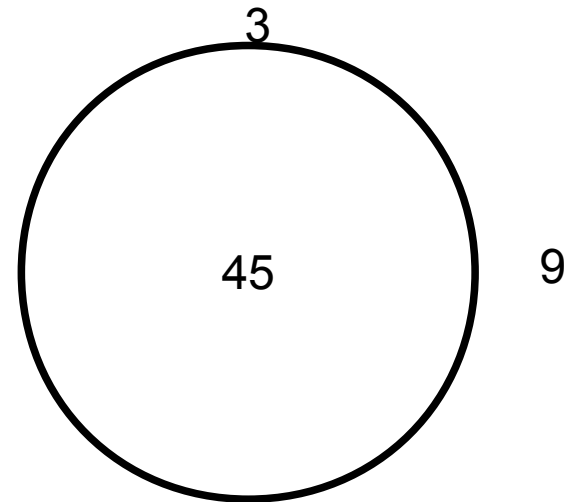
# YBC 7302



La loro posizione indica il significato:

3 si riferisce al contorno  
45 all'"interno"

Che numeri?  
Che relazione fra loro?



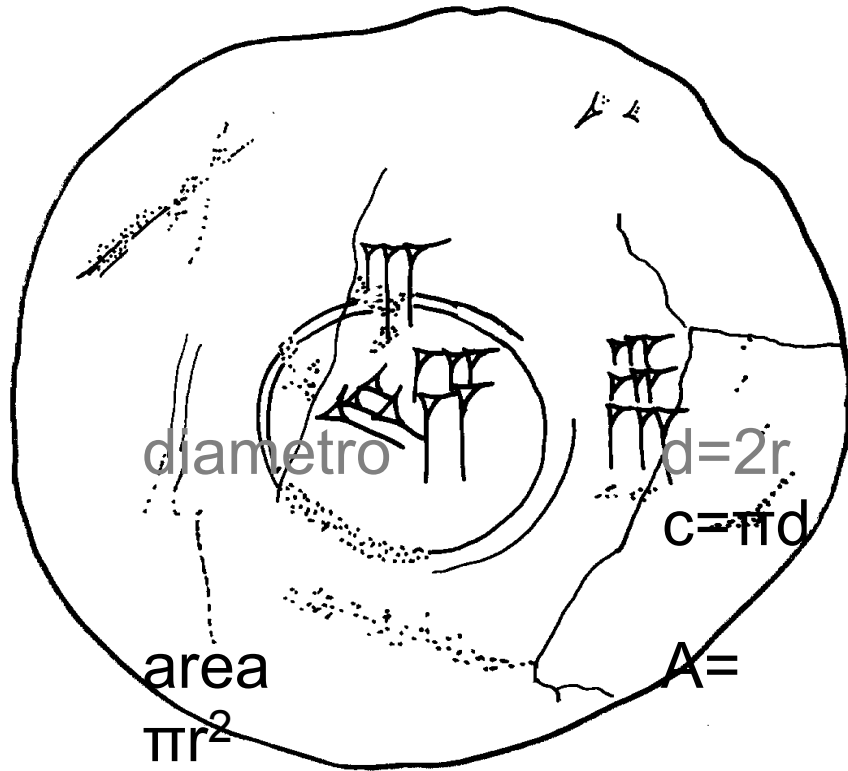
$$9 = 3 \times 3$$

$$45 = 9 \times 5$$

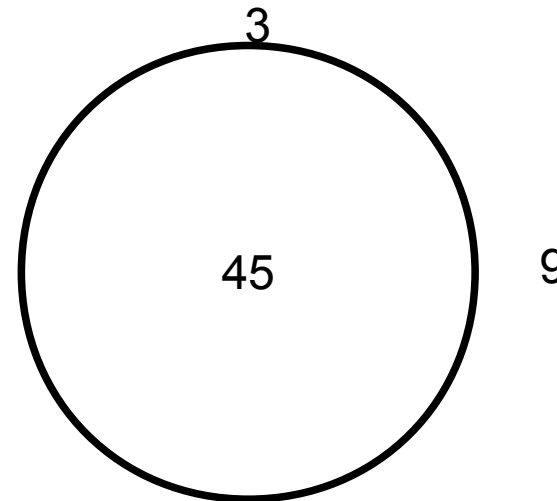
$$[;45] = [9] \times [;5]$$

$$9 \times \frac{1}{12} = \frac{9}{12} = \frac{3}{4} = \frac{45}{60}$$

# YBC 7302



Che numeri?  
Che relazione fra loro?



Come avremmo fatto noi:

$$c = 2\pi r \quad r = c/2\pi$$

$$r = 3/2\pi = 3/2 \cdot 3,14 = 0,4777$$

$$A = \pi r^2 \quad A = \pi 0,4777^2 = 0,71619$$

oppure

$$A = \pi (c/2\pi)^2 = c^2/4\pi = 9/4\pi = 0,71619$$

$$9 = 3 \times 3$$

$$45 = 9 \times 5$$

$$[:45] = [9] \times [:5]$$

$$\begin{aligned} 9 \times 1/12 &= 9/12 = \\ 3/4 &= 45/60 \\ &= 0,75 \end{aligned}$$

# CIRCONFERENZA e CERCHIO

## Le “nostre” formule

circonferenza	$c=2\pi r$
diametro	$d=2r$ $c=\pi d$
area	$A=\pi r^2$ $A=c^2/4\pi$

## Le formule babilonesi

circonferenza	$c$
area	$A=[0; 05] \times c^2$ $A=1/12 c^2$
diametro	$d=c/3$



“contorno” misura fondamentale

molti esempi; l'area è sempre calcolata a partire dalla circonferenza, anche quando si conosce il diametro

la parola “kippatum” (cosa che curva) indica sia il cerchio (“pieno”) che il bordo

# CIRCONFERENZA e CERCHIO

Le "nostre" formule

circonferenza  $c=2\pi r$   
diametro  $d=2r$   
 $c=\pi d$

area  $A= \pi r^2$

La corrispondenza

$$c=2\pi r \rightarrow r=c/2\pi \rightarrow r^2=c^2/4\pi^2$$

$$A= c^2/4\pi$$

Le formule babilonesi

circonferenza  $c$

area  $A=[0; 05] \times c^2$   
 $A= c^2 /12$   
 $A= c^2 \cdot 1/12$

diametro  $d=c/3$

$$A= c^2 \cdot 1/4 \cdot 3$$

$\pi$  corrisponde a 3

... qualche altro caso:  $\pi \approx 3.1$

## CIRCONFERENZA e CERCHIO

Un triangolo equilatero inscritto in un cerchio

Problema più avanzato  
figura estremamente precisa

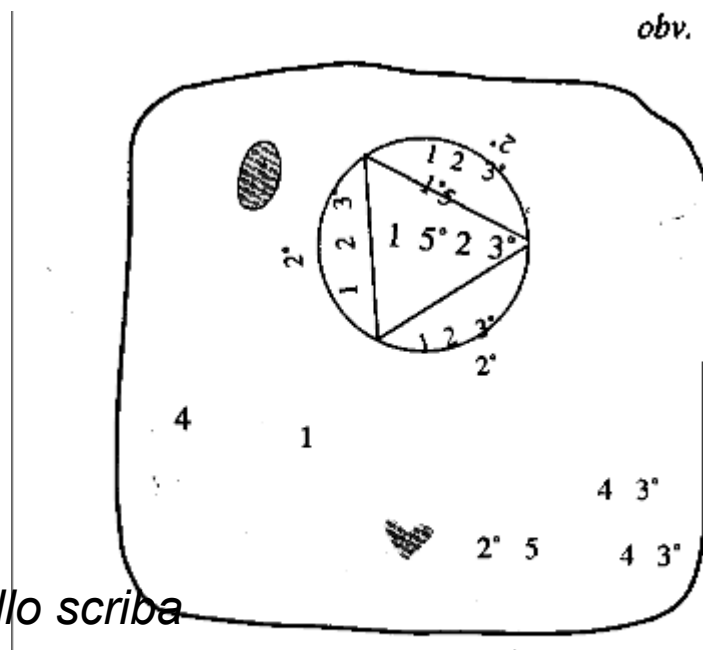




# CIRCONFERENZA e CERCHIO

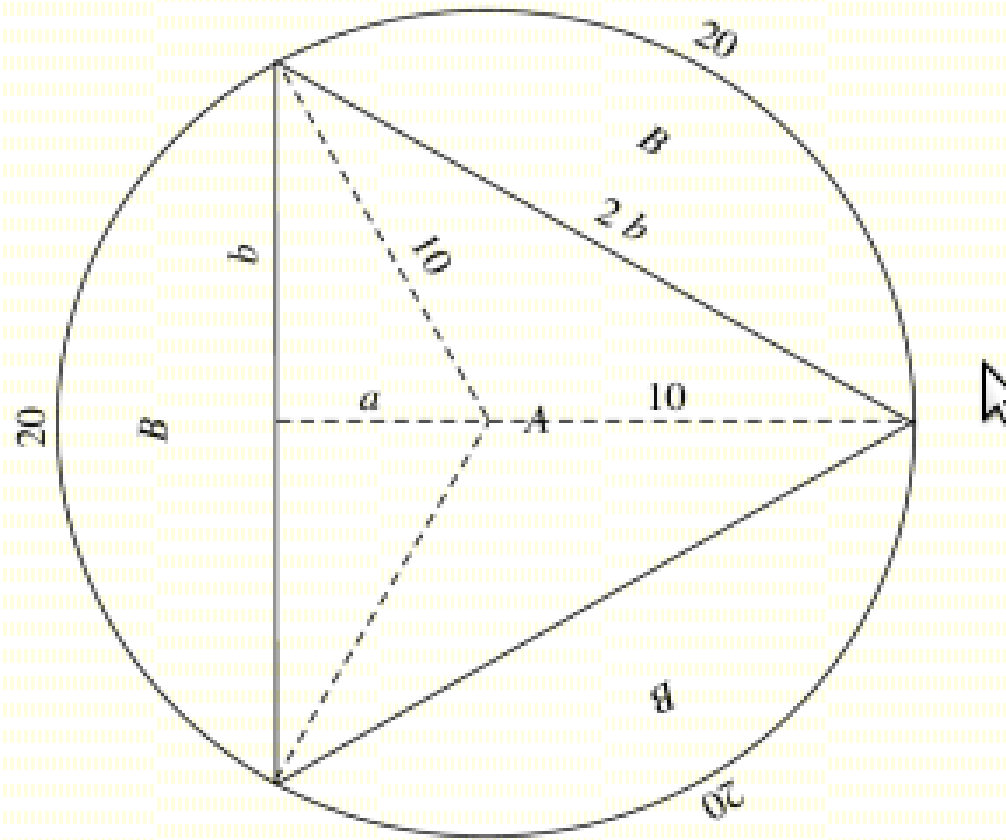
Un triangolo equilatero inscritto in un cerchio

Problema più avanzato  
figura estremamente precisa



la circonferenza è  $c=1\ 00$  (ninda)  
 ciascun arco è 20 (ninda)  $20/60=1/3$   
 il diametro è  $d=c/3$  quindi 20, il raggio 10  
 l'area del cerchio  $C=1/12\ c^2 = 0; 05\ c^2 = 5\ 00$

l'area del triangolo A?  
 L'area dei segmenti circolari B?



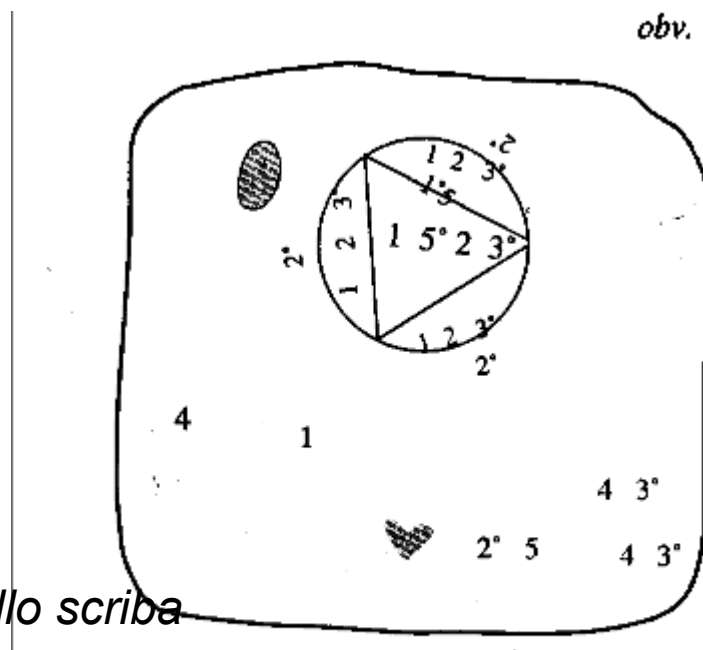
Come ha ottenuto l'area dei segmenti circolari?

$$B = (C-A) : 3 = (5\ 00 - 1\ 52;30) : 3 = 1\ 02;30$$

# CIRCONFERENZA e CERCHIO

Un triangolo equilatero inscritto in un cerchio

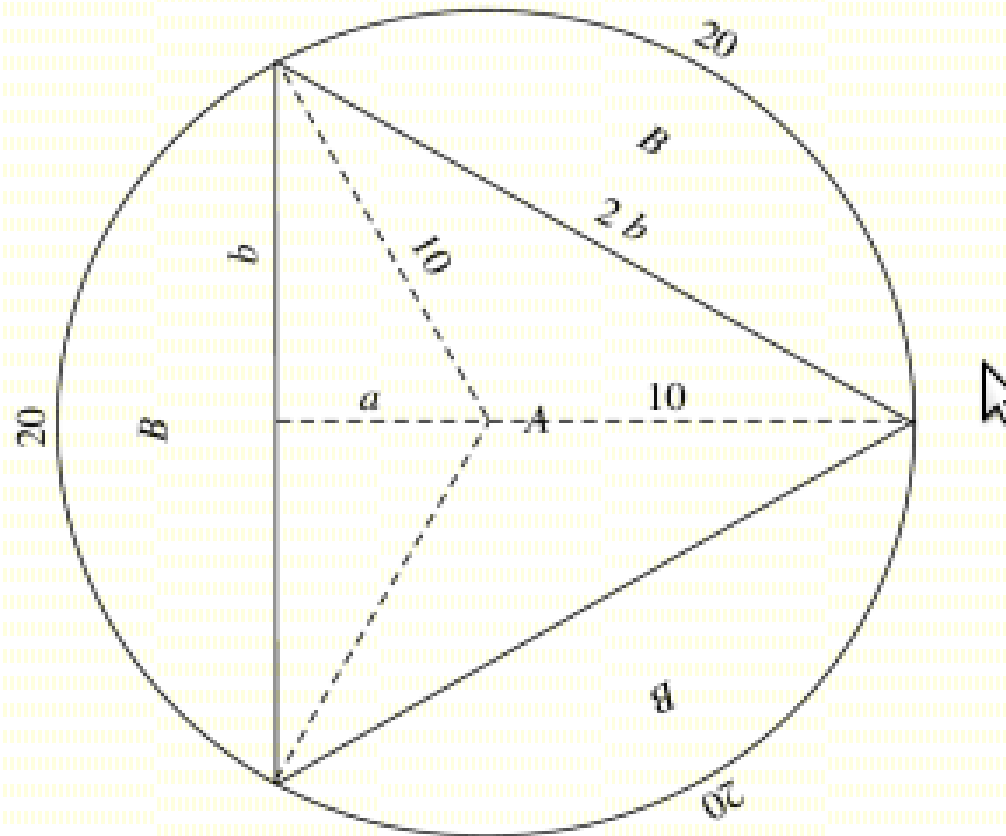
Problema più avanzato  
figura estremamente precisa



*I calcoli dello scriba*

la circonferenza è  $c=100$  (ninda)  
 ciascun arco è  $20$  (ninda)  $20/60=1/3$   
 il diametro è  $d=c/3$  quindi  $20$ , il raggio  $10$   
 l'area del cerchio  $C=1/12 c^2 = 0; 05 c^2 = 500$

l'area del triangolo A?  
 L'area dei segmenti circolari B?



Come ha ottenuto l'area del triangolo?

$15230$  si ottiene come  $15 \times 15 / 2$

Errore!

$h=r+a=10+5=15$

Ma non trova b!

Prende la base del triangolo uguale all'altezza!

$$b:a=(a+r):b$$

$$b:5=15:b$$



# DIAGONALI E TEOREMA DI PITAGORA?

Yale Babylonian Collection YBC 7289



recto



verso

## DIAGONALI E TEOREMA DI PITAGORA?

Yale Babylonian Collection YBC 7289



recto

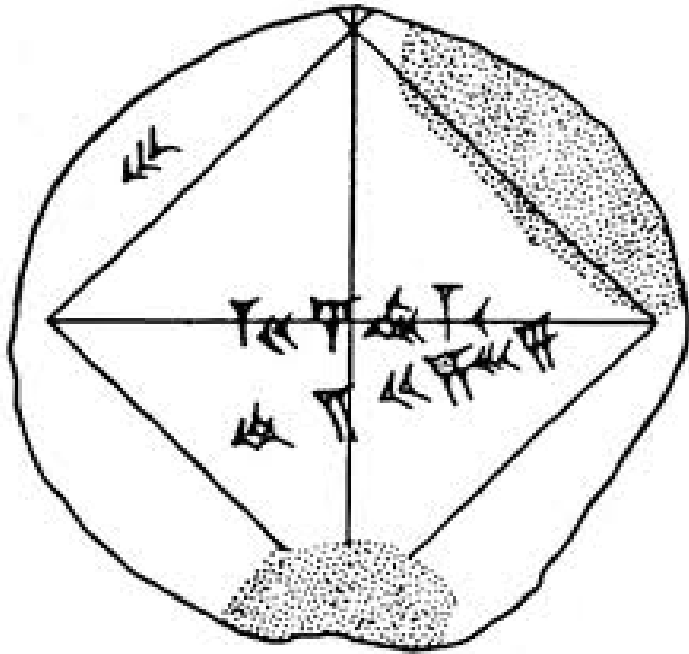
La prima testimonianza nota relativa al  
teorema di Pitagora

databile tra il 1800 e il 1600 a. C. (periodo  
paleobabilonese)

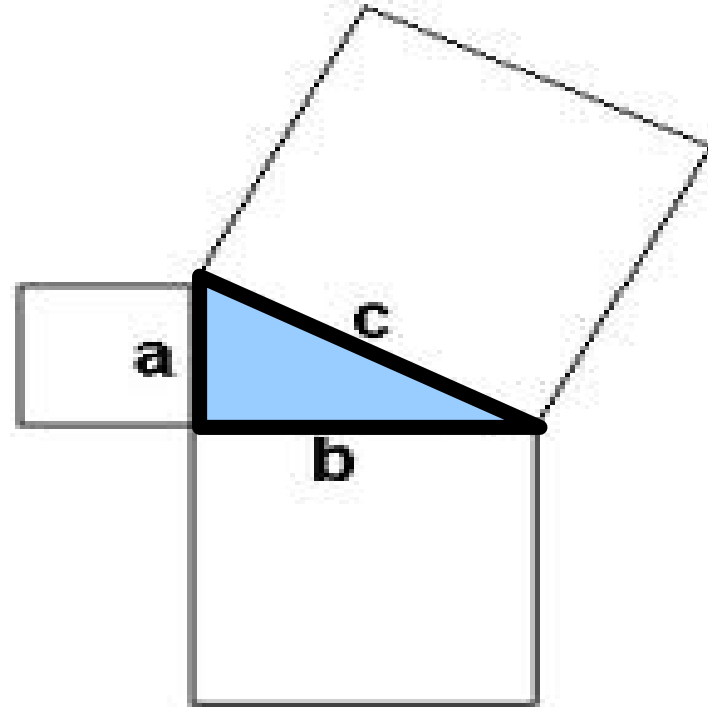
Mesopotamia meridionale

# DIAGONALI E TEOREMA DI PITAGORA?

Yale Babylonian Collection YBC 7289



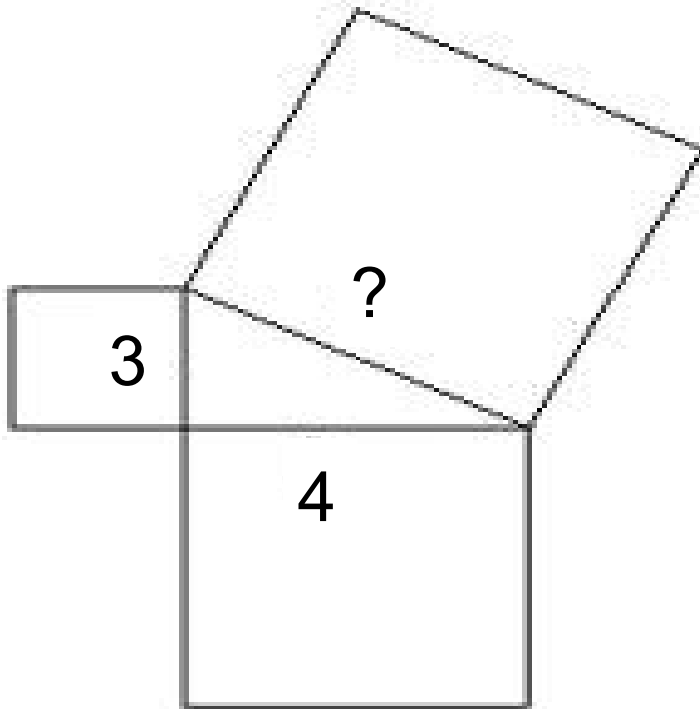
Teorema di Pitagora



$$a^2 + b^2 = c^2$$

# DIAGONALI E TEOREMA DI PITAGORA?

Yale Babylonian Collection YBC 7289

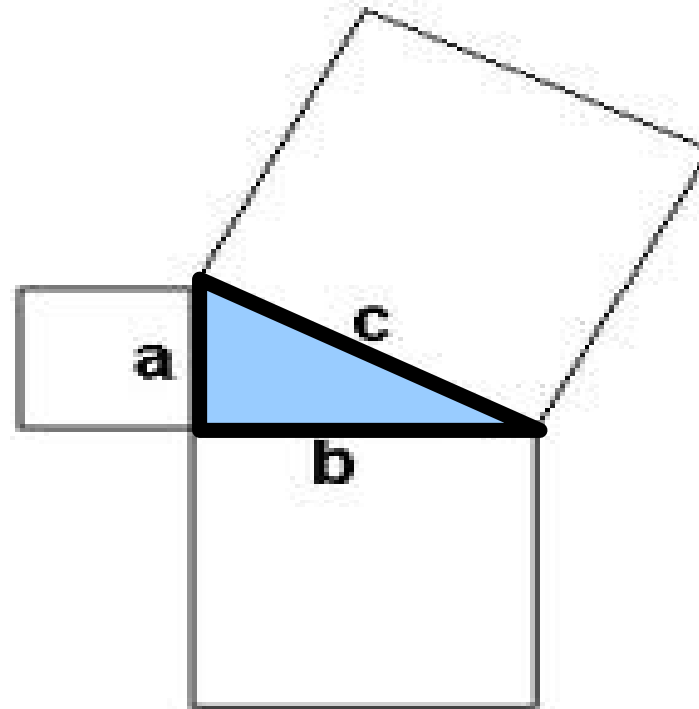


$$3^2 + 4^2 = c^2$$

$$9 + 16 = 25$$

$$c^2 = 25 \quad c = 5$$

Teorema di Pitagora

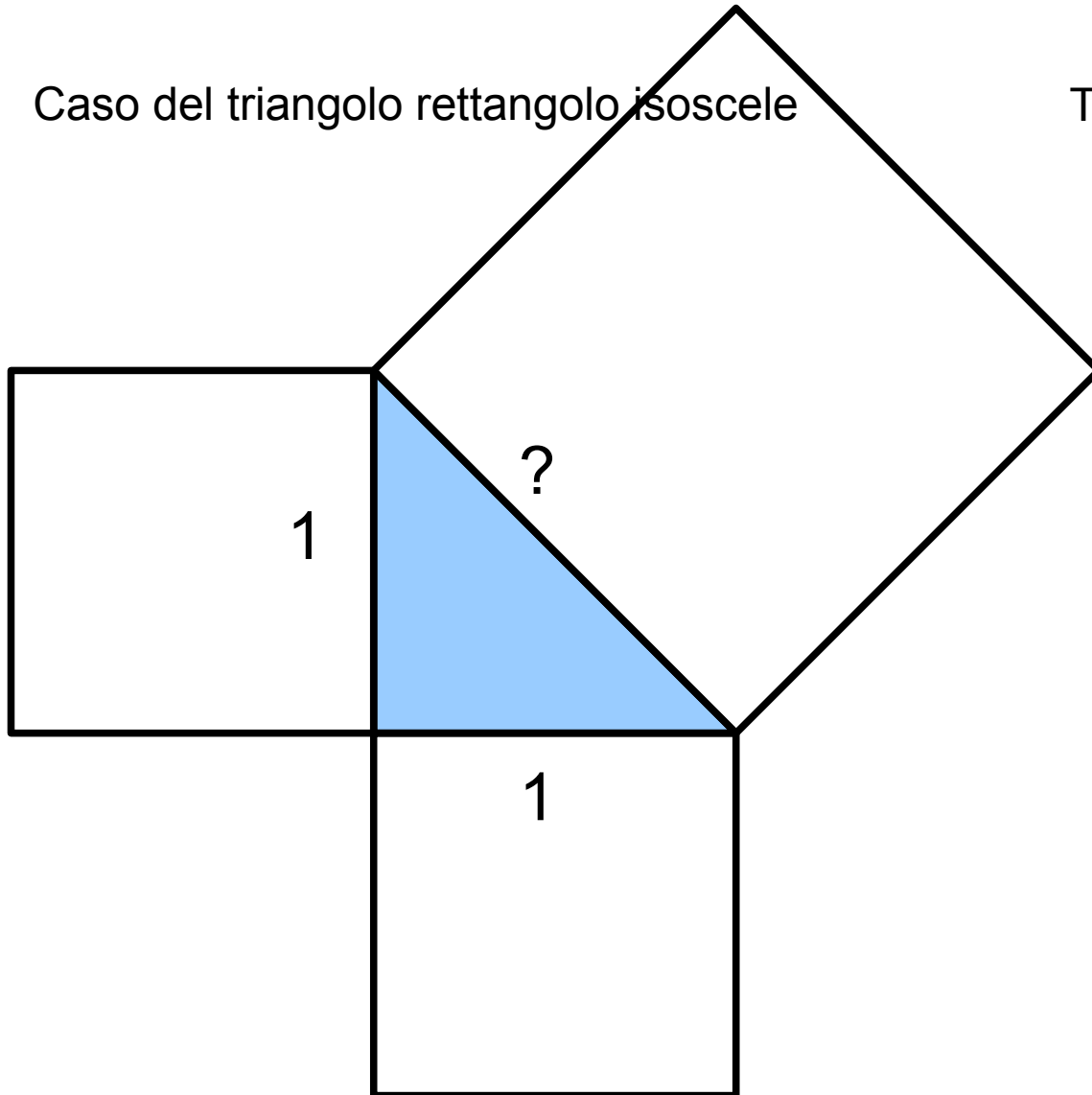


$$a^2 + b^2 = c^2$$

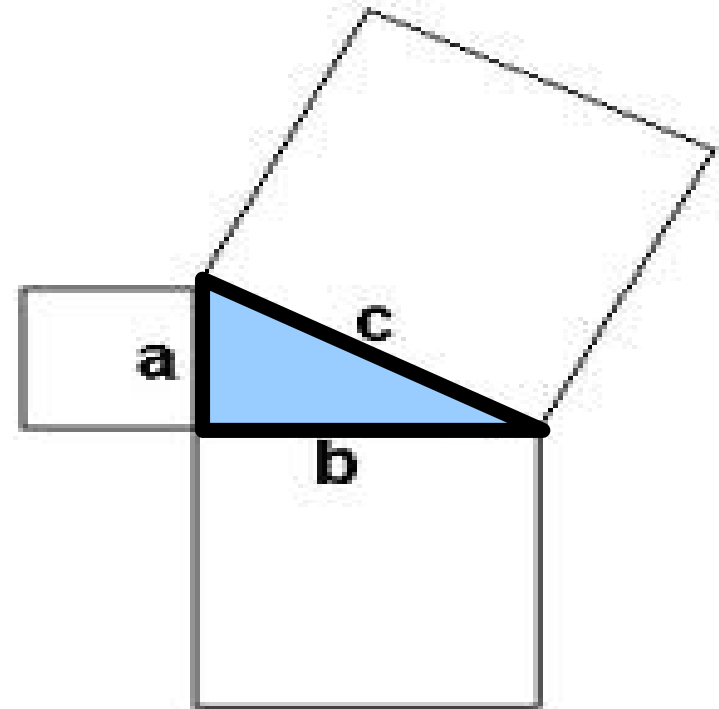
# DIAGONALI E TEOREMA DI PITAGORA?

Yale Babylonian Collection YBC 7289

Caso del triangolo rettangolo isoscele



Teorema di Pitagora



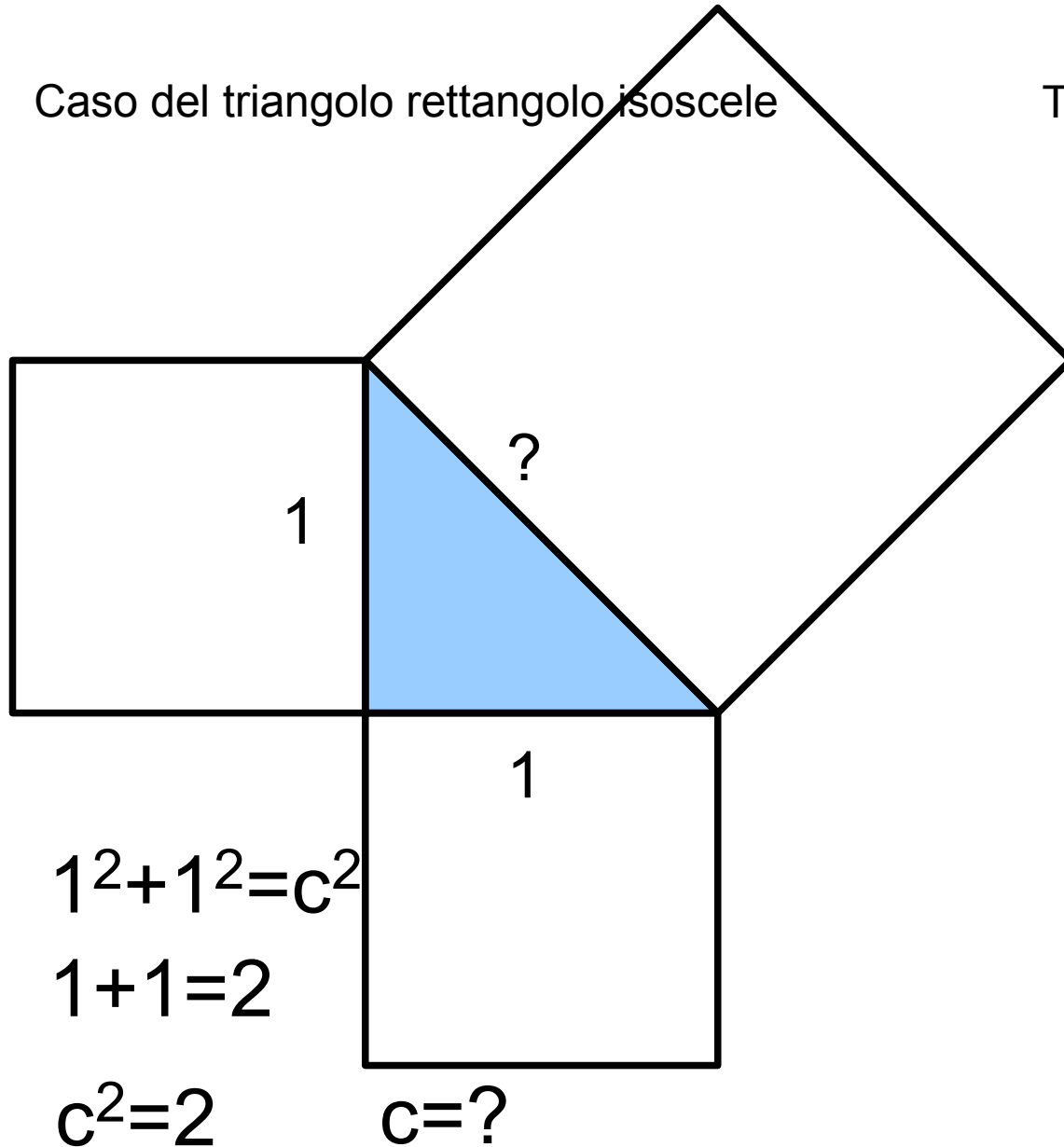
$$a^2 + b^2 = c^2$$



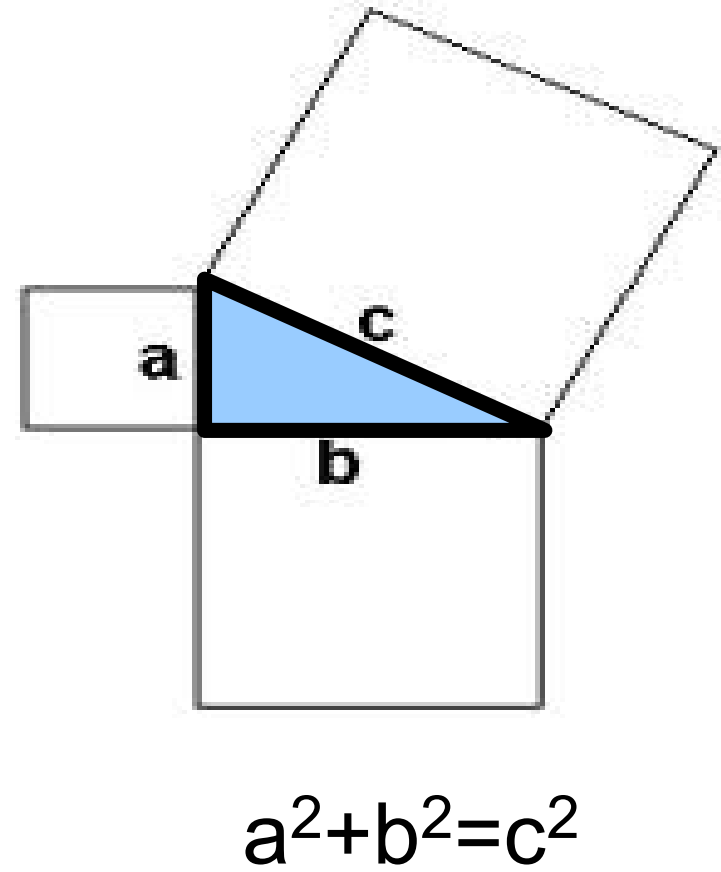
# DIAGONALI E TEOREMA DI PITAGORA?

Yale Babylonian Collection YBC 7289

Caso del triangolo rettangolo isoscele



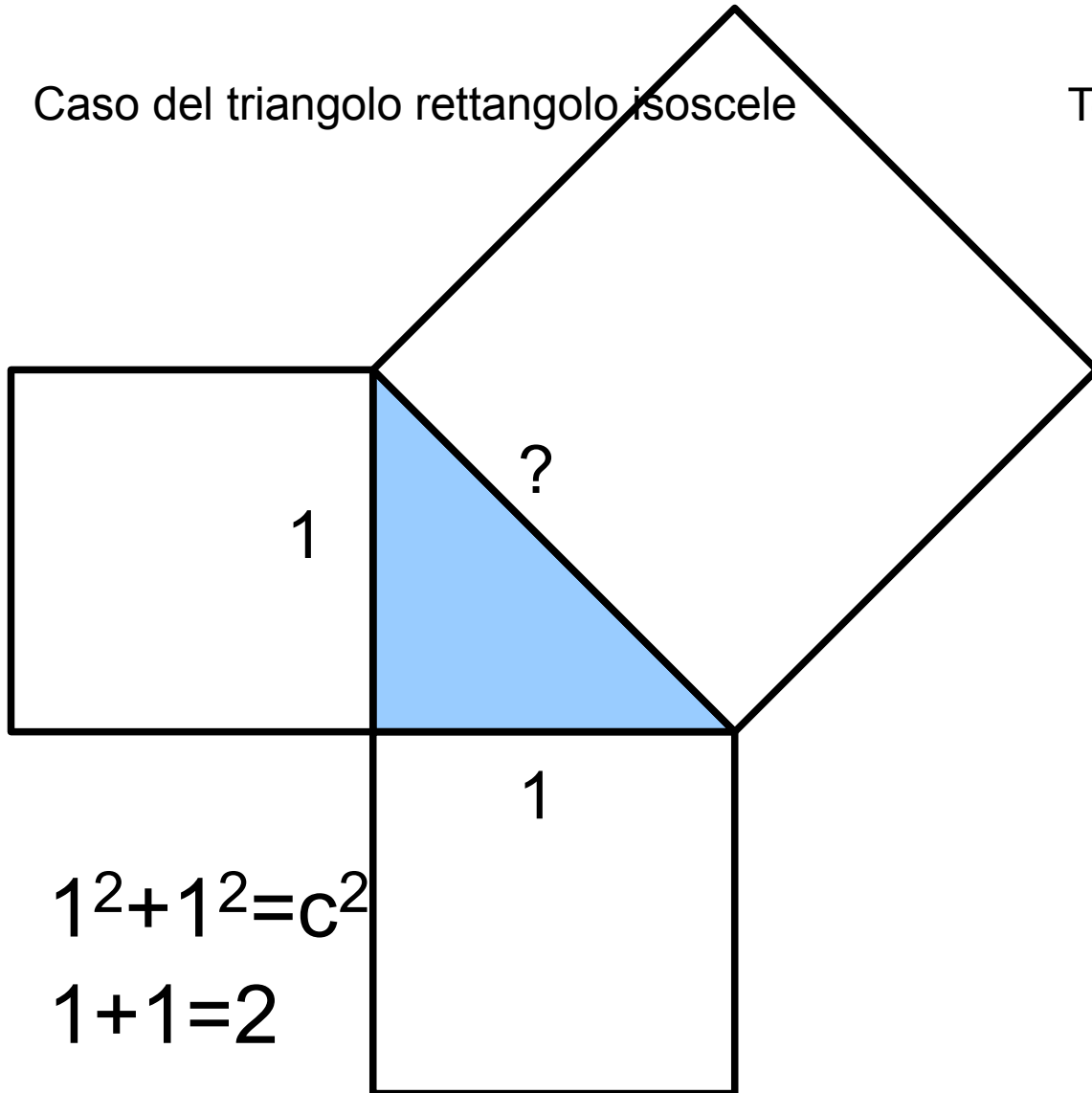
Teorema di Pitagora



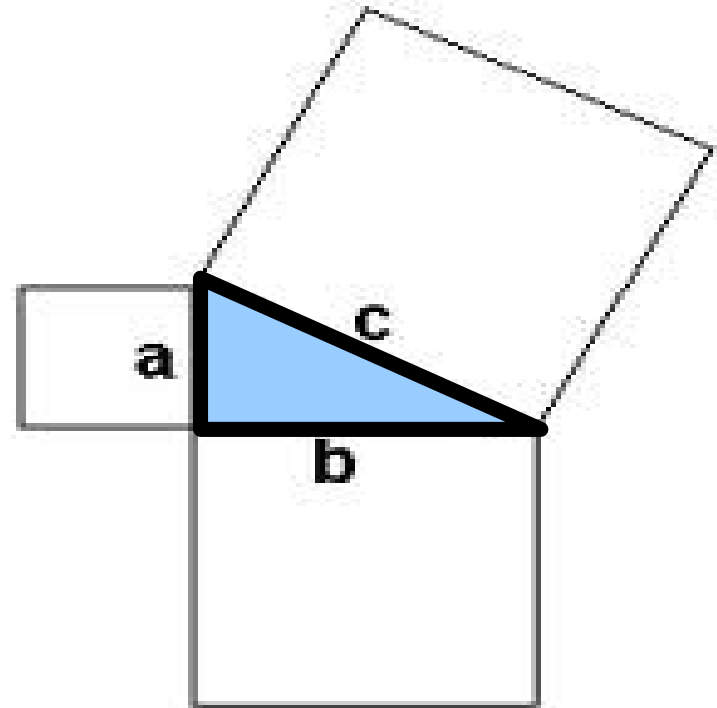
# DIAGONALI E TEOREMA DI PITAGORA?

Yale Babylonian Collection YBC 7289

Caso del triangolo rettangolo isoscele



Teorema di Pitagora



$$1^2 + 1^2 = c^2$$

$$1 + 1 = 2$$

$$c^2 = 2$$

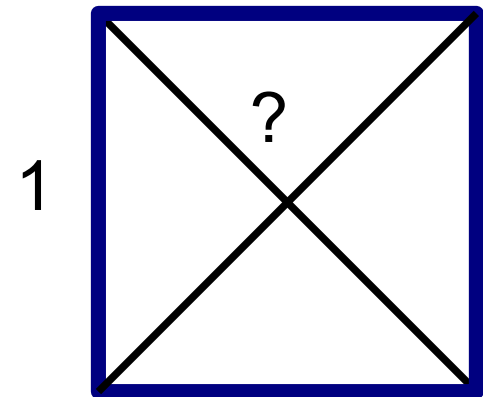
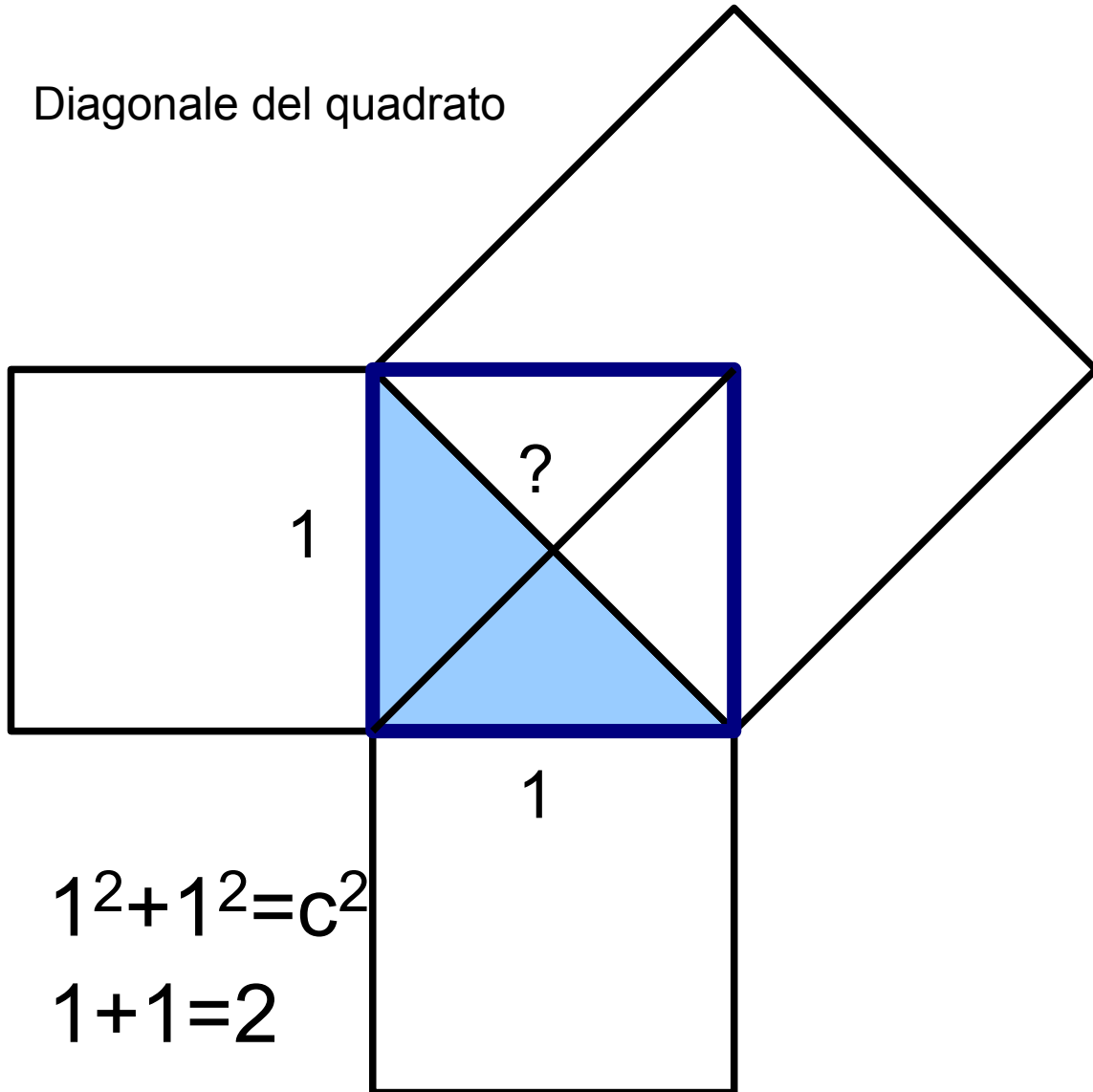
$$C = \sqrt{2} = 1, 414213562373095048801...$$

$$a^2 + b^2 = c^2$$

# DIAGONALI E TEOREMA DI PITAGORA?

Yale Babylonian Collection YBC 7289

Diagonale del quadrato



$$1^2 + 1^2 = c^2$$

$$1 + 1 = 2$$

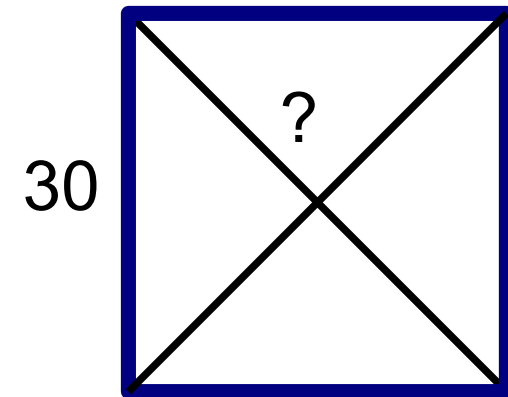
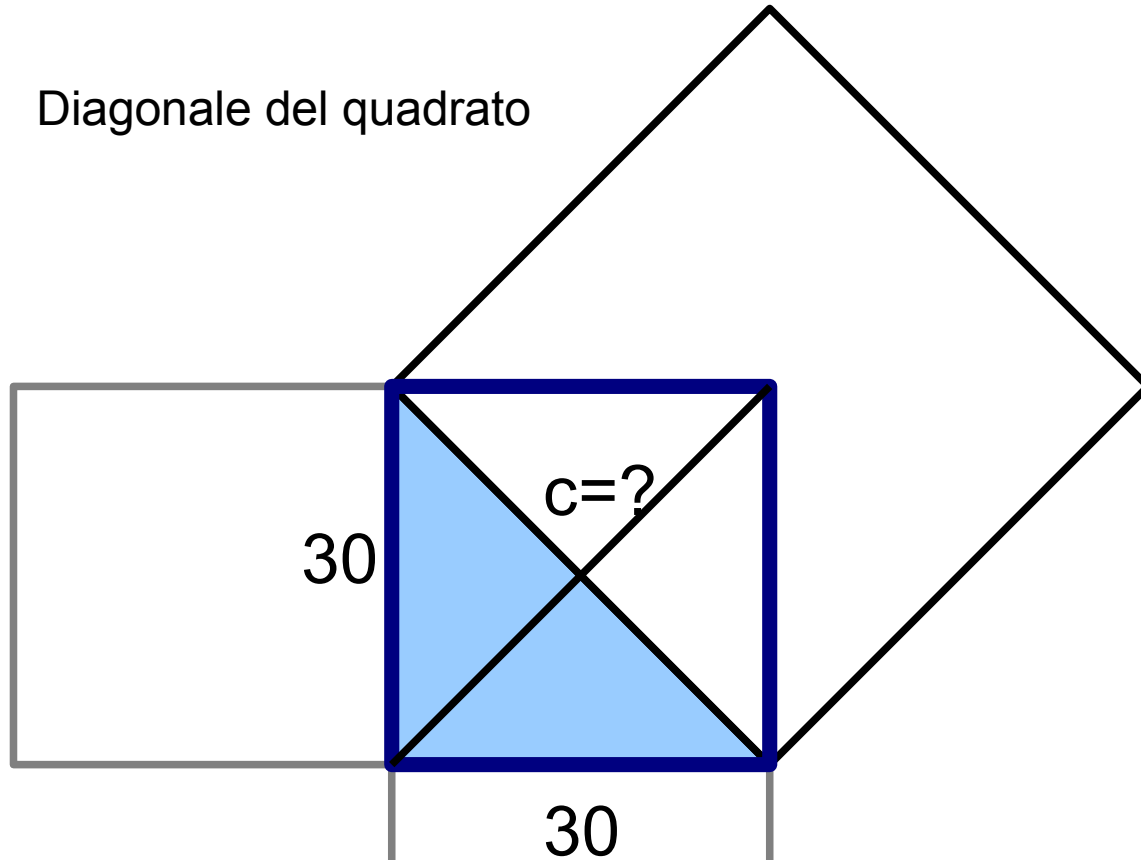
$$c^2 = 2$$

$$C = \sqrt{2} = 1, 414213562373095048801...$$

# DIAGONALI E TEOREMA DI PITAGORA?

Yale Babylonian Collection YBC 7289

Diagonale del quadrato

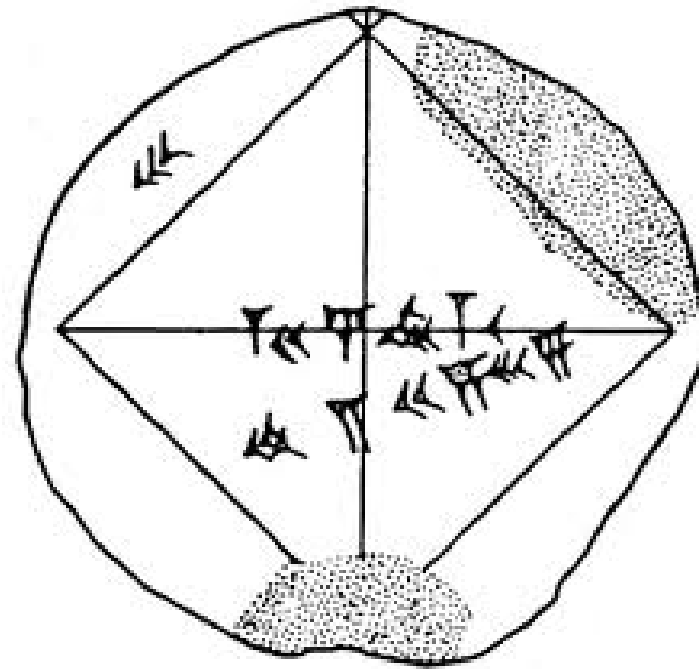
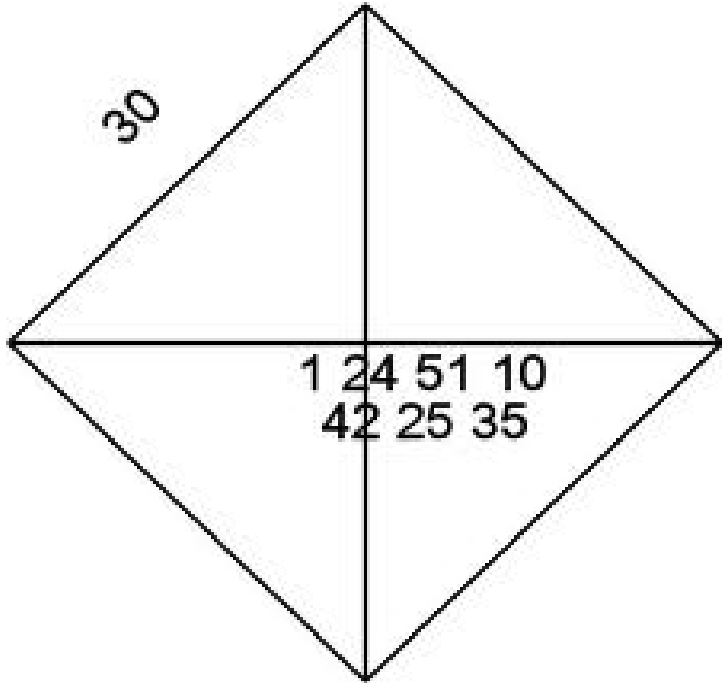


$$30^2 + 30^2 = c^2$$

$$c = \sqrt{900 + 900} = \sqrt{900 \times 2} = \sqrt{1800} = 42,42639 \dots$$
$$= \sqrt{900 \times 2} = 30 \times \sqrt{2} = 42,42639 \dots$$

# DIAGONALI E TEOREMA DI PITAGORA?

Yale Babylonian Collection YBC 7289



1;24,51,10,

Che relazione tra questi numeri?

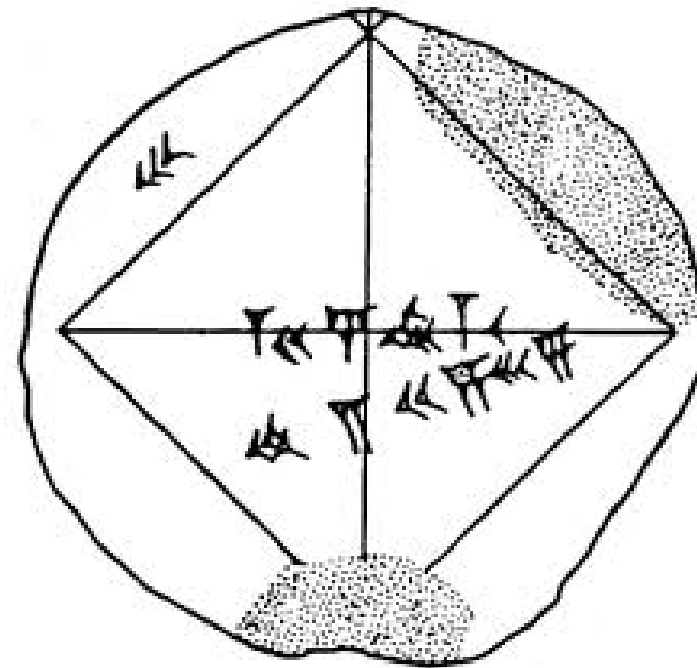
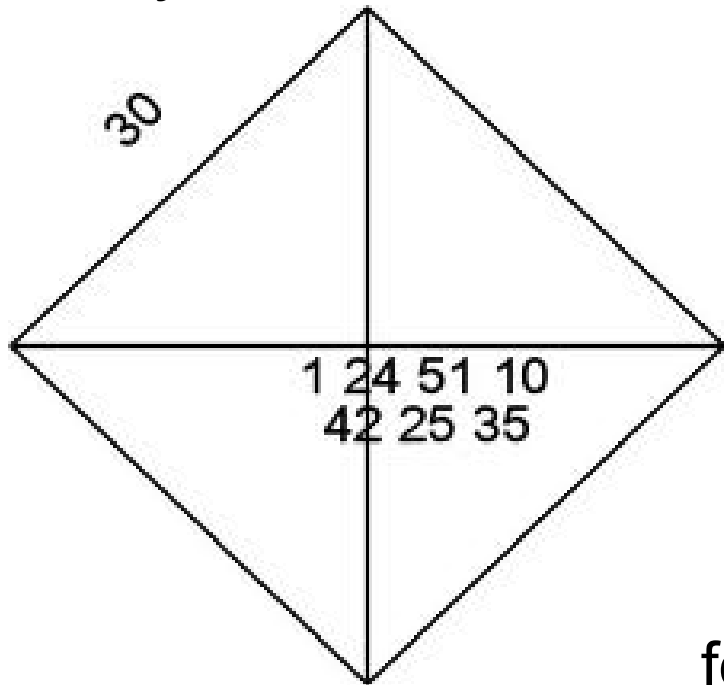
42;25,35,

30

$$1;24,51,10 \times 30 = 42;25,35$$

# DIAGONALI E TEOREMA DI PITAGORA?

Yale Babylonian Collection YBC 7289



forma  
decimale

1;24,51,10,

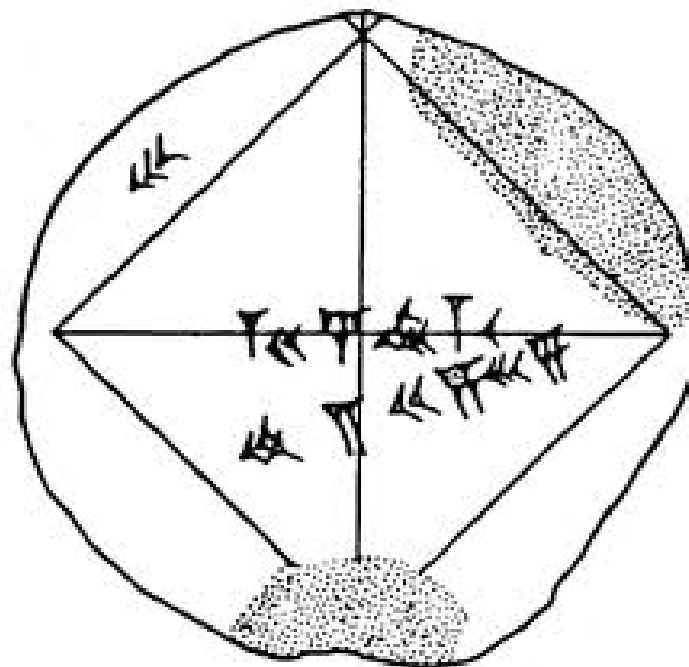
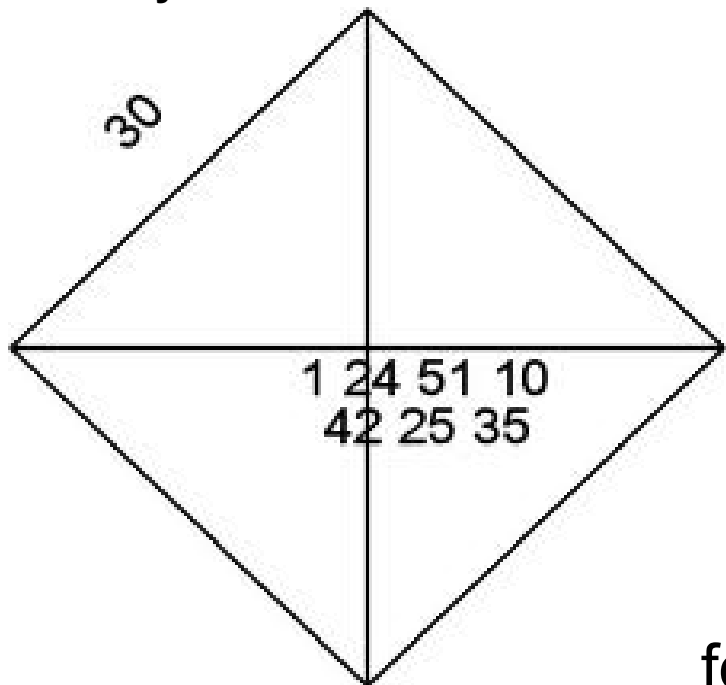
$$1 + 24/60 + 51/60^2 + 10/60^3$$

42;25,35,

$$42 + 25/60 + 35/60^2$$

# DIAGONALI E TEOREMA DI PITAGORA?

Yale Babylonian Collection YBC 7289



forma  
decimale

1;24,51,10,

$$1 + 24/60 + 51/60^2 + 10/60^3$$

1,41421

42;25,35,

$$42 + 25/60 + 35/60^2$$

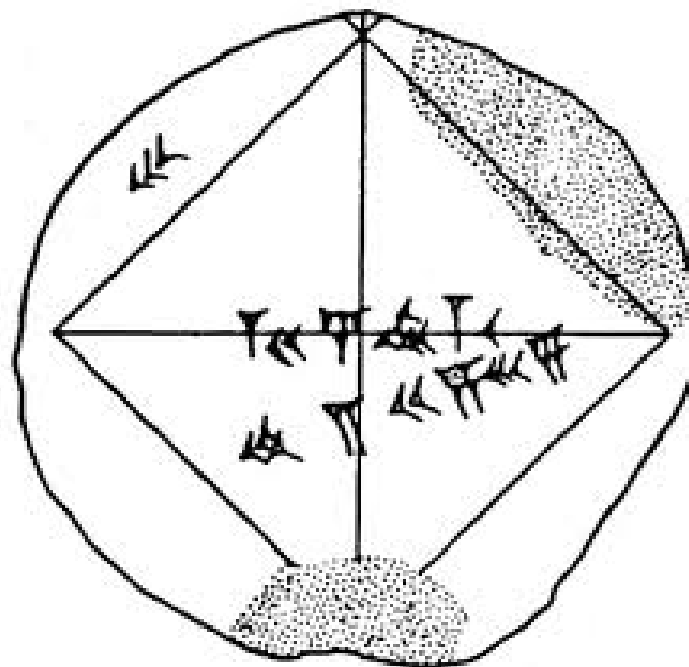
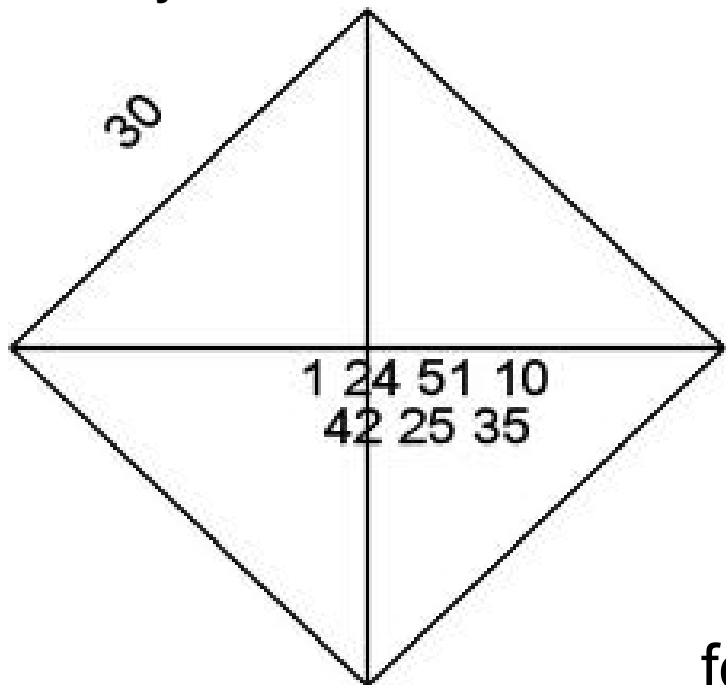
<sup>3</sup>42,42639

Il primo è un'ottima approssimazione della radice di 2;

il secondo è la diagonale del quadrato di lato 30, ed è uguale al prodotto di 30 per il primo numero.

# DIAGONALI E TEOREMA DI PITAGORA?

Yale Babylonian Collection YBC 7289



forma  
decimale

1;24,51,10,

$$1 + 24/60 + 51/60^2 + 10/60^3$$

1,41421

42;25,35,

$$42 + 25/60 + 35/60^2$$

<sup>3</sup>42,42639

conoscenza del teorema di Pitagora, almeno nel caso del triangolo con i cateti uguali ?

Questa tavoletta da sola non dimostra che i Babilonesi conoscessero il “teorema di Pitagora” nella sua generalità, ma esistono altre tavolette ...



Plimpton 322 (1800 a.C. circa)



# Plimpton 322 (1800 a.C. circa)

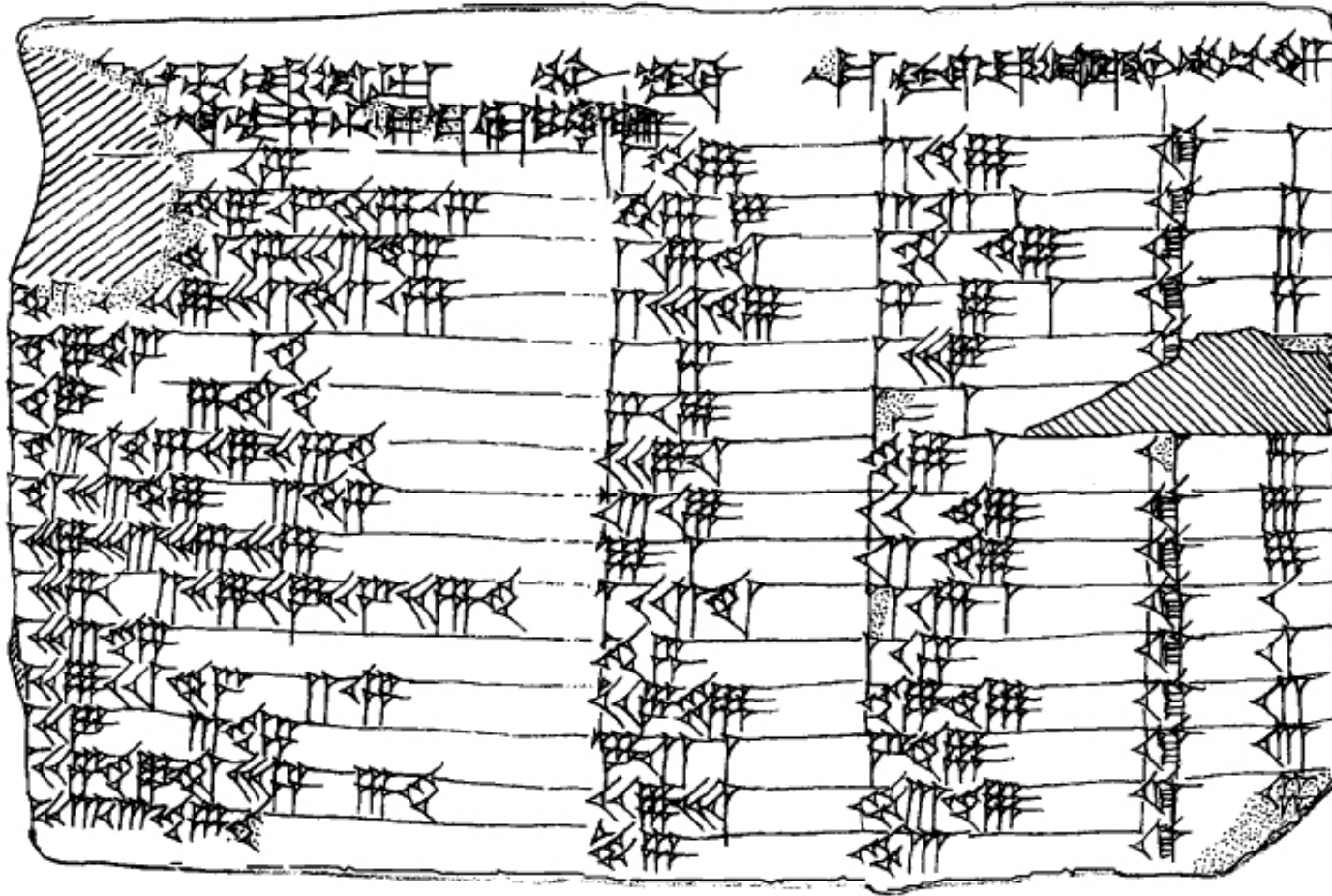


tabella  
quattro colonne di numeri  
quindici righe

quarta colonna:  
lista di numeri da 1 a 15

seconda e terza colonna  
sono completamente visibili

angolo della prima colonna  
scheggiato

Figure 1. Plimpton 322 (obverse). Drawing by the author.

## Plimpton 322 (1800 a.C. circa)

(1:)59:00:15	1:59	2:49	1
(1:)56:56:58:14:50:06:15	56:07	1:20:25	2
(1:)55:07:41:15:33:45	1:16:41	1:50:49	3
(1:)53:10:29:32:52:16	3:31:49	5:09:01	4
(1:)48:54:01:40	1:05	1:37	5
(1:)47:06:41:40	5:19	8:01	6
(1:)43:11:56:28:26:40	38:11	59:01	7
(1:)41:33:45:14:03:45	13:19	20:49	8
(1:)38:33:36:36	8:01	12:49	9
(1:)35:10:02:28:27:24:26	1:22:41	2:16:01	10
(1:)33:45	45	1:15	11
(1:)29:21:54:02:15	27:59	48:49	12
(1:)27:00:03:45	2:41	4:49	13
(1:)25:48:51:35:06:40	29:31	53:49	14
(1:)23:13:46:40	56	1:46	15

tabella  
quattro colonne di numeri  
quindici righe

quarta colonna:  
lista di numeri da 1 a 15

seconda e terza colonna  
sono completamente visibili

angolo della prima colonna  
scheggiato

# Plimpton 322 (1800 a.C. circa)

colonna 1  
 $b^2/a^2$

col. 2  
b

col.3  
c

(1)59:00:15	1:59	2:49	1
(1)56:56:58:14:50:06:15	56:07	1:20:25	2
(1)55:07:41:15:33:45	1:16:41	1:50:49	3
(1)53:10:29:32:52:16	3:31:49	5:09:01	4
(1)48:54:01:40	1:05	1:37	5
(1)47:06:41:40	5:19	8:01	6
(1)43:11:56:28:26:40	38:11	59:01	7
(1)41:33:45:14:03:45	13:19	20:49	8
(1)38:33:36:36	8:01	12:49	9
(1)35:10:02:28:27:24:26	1:22:41	2:16:01	10
(1)33:45	45	1:15	11
(1)29:21:54:02:15	27:59	48:49	12
(1)27:00:03:45	2:41	4:49	13
(1)25:48:51:35:06:40	29:31	53:49	14
(1)23:13:46:40	56	1:46	15

## Interpretazioni

terne pitagoriche,  
terne di numeri interi  
 $a^2+b^2=c^2$

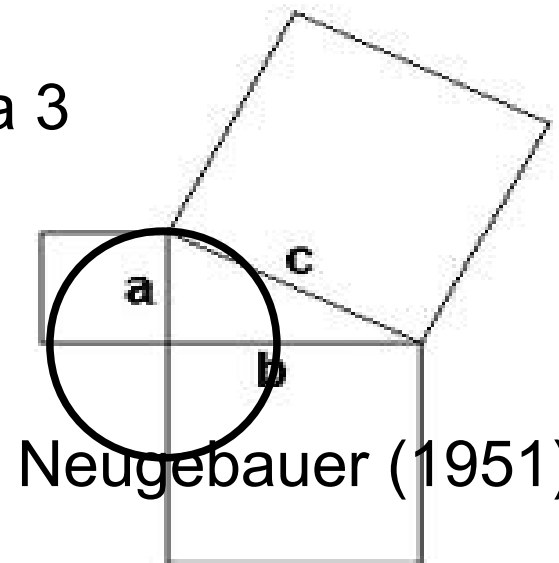
colonna 1  
 $b^2/a^2$

colonna 2

b

colonna 3

c





# Plimpton 322 (1800 a.C. circa)

colonna 1  
 $b^2/a^2$

col. 2  
b

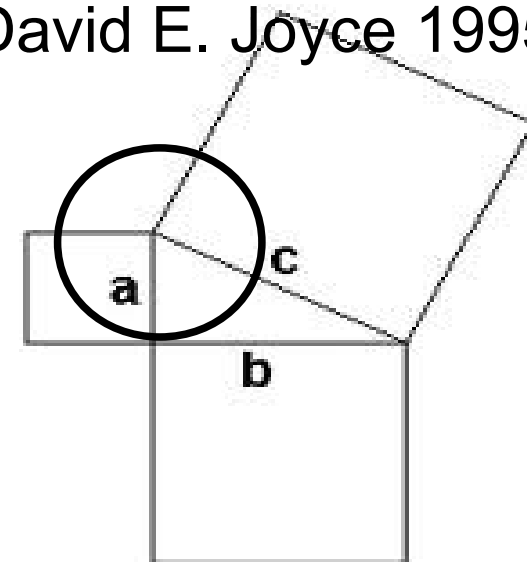
col.3  
c

(1:)59:00:15	1:59	2:49	1
(1:)56:56:58:14:50:06:15	56:07	1:20:25	2
(1:)55:07:41:15:33:45	1:16:41	1:50:49	3
(1:)53:10:29:32:52:16	3:31:49	5:09:01	4
(1:)48:54:01:40	1:05	1:37	5
(1:)47:06:41:40	5:19	8:01	6
(1:)43:11:56:28:26:40	38:11	59:01	7
(1:)41:33:45:14:03:45	13:19	20:49	8
(1:)38:33:36:36	8:01	12:49	9
(1:)35:10:02:28:27:24:26	1:22:41	2:16:01	10
(1:)33:45	45	1:15	11
(1:)29:21:54:02:15	27:59	48:49	12
(1:)27:00:03:45	2:41	4:49	13
(1:)25:48:51:35:06:40	29:31	53:49	14
(1:)23:13:46:40	56	1:46	15

## Interpretazioni

tavola trigonometrica di quadrati di cosecanti che vanno da  $45^\circ$  fino a  $30^\circ$

David E. Joyce 1995

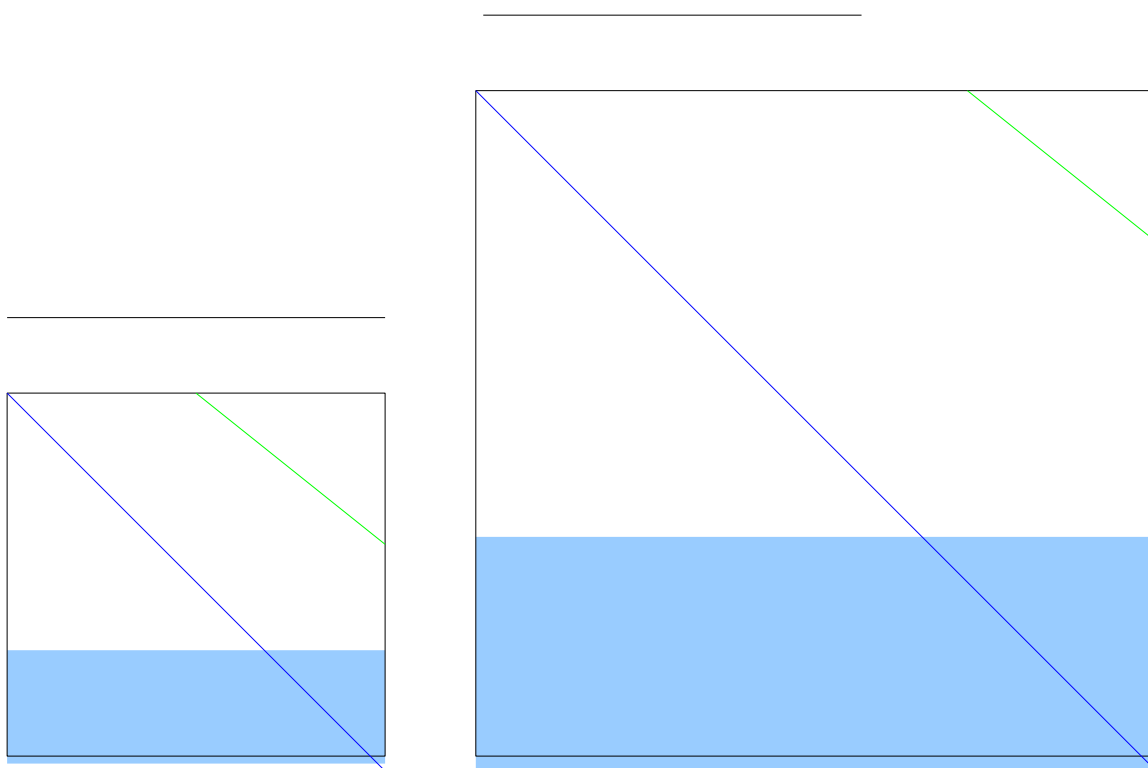


I babilonesi conoscevano l'"angolo"?

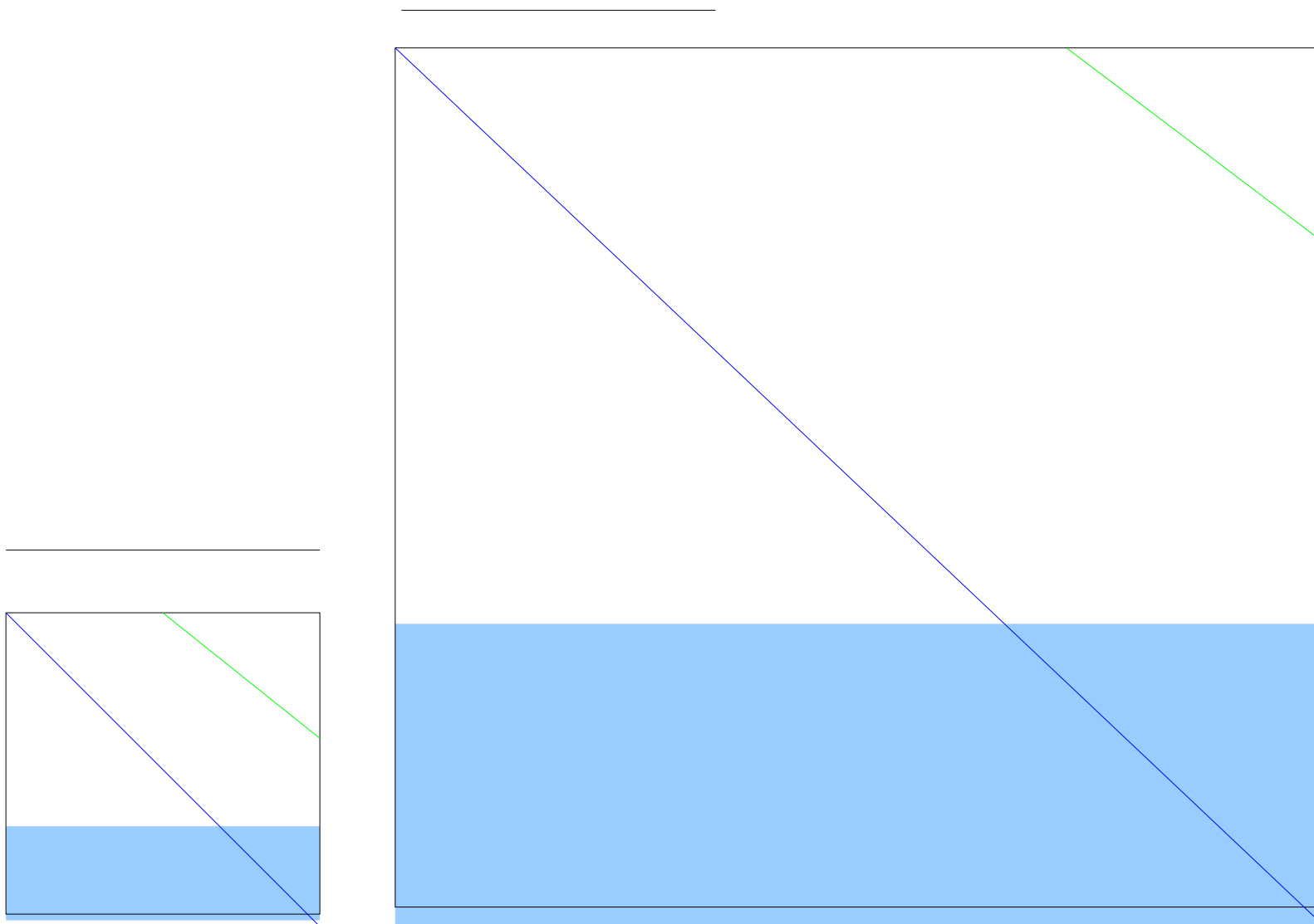
# PROPORZIONI E SIMILITUDINE

anche in geometria

- in una configurazione di forma ben definita
    - le lunghezze sono proporzionali
    - le aree sono proporzionali al quadrato di una dimensione lineare
- > tabelle di “costanti tecniche”



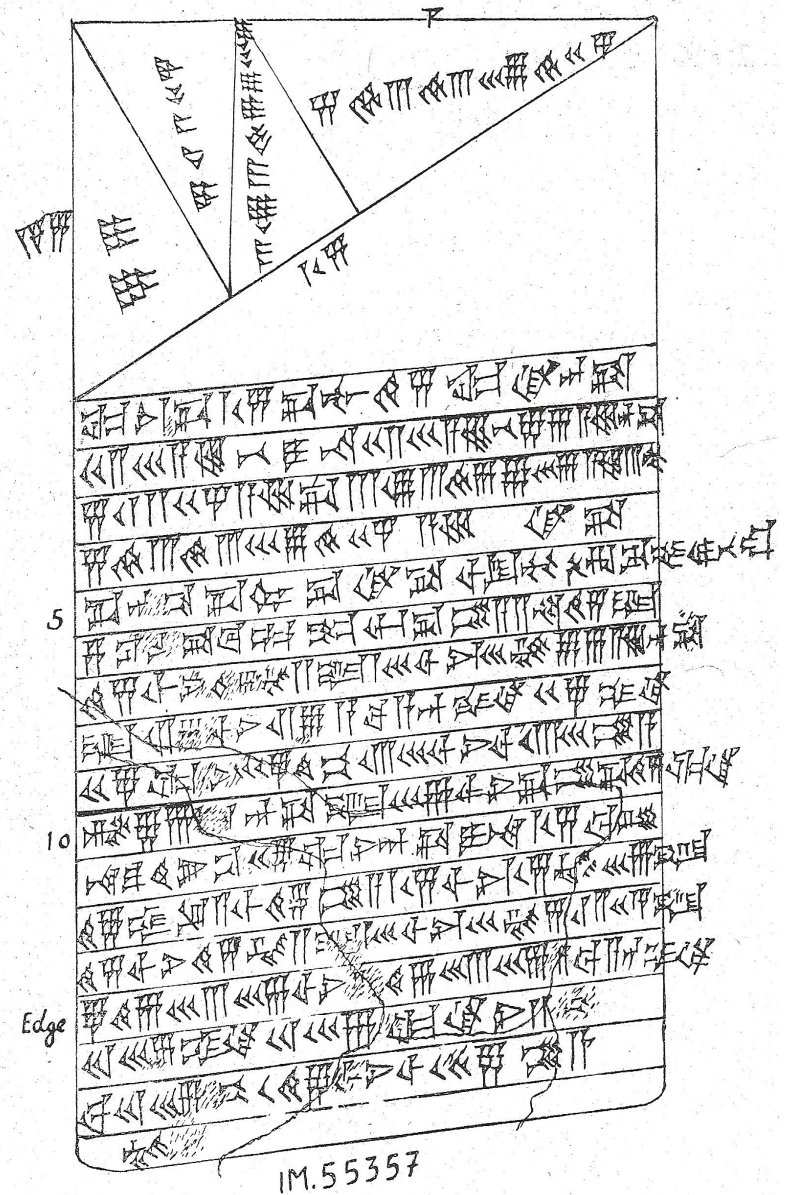
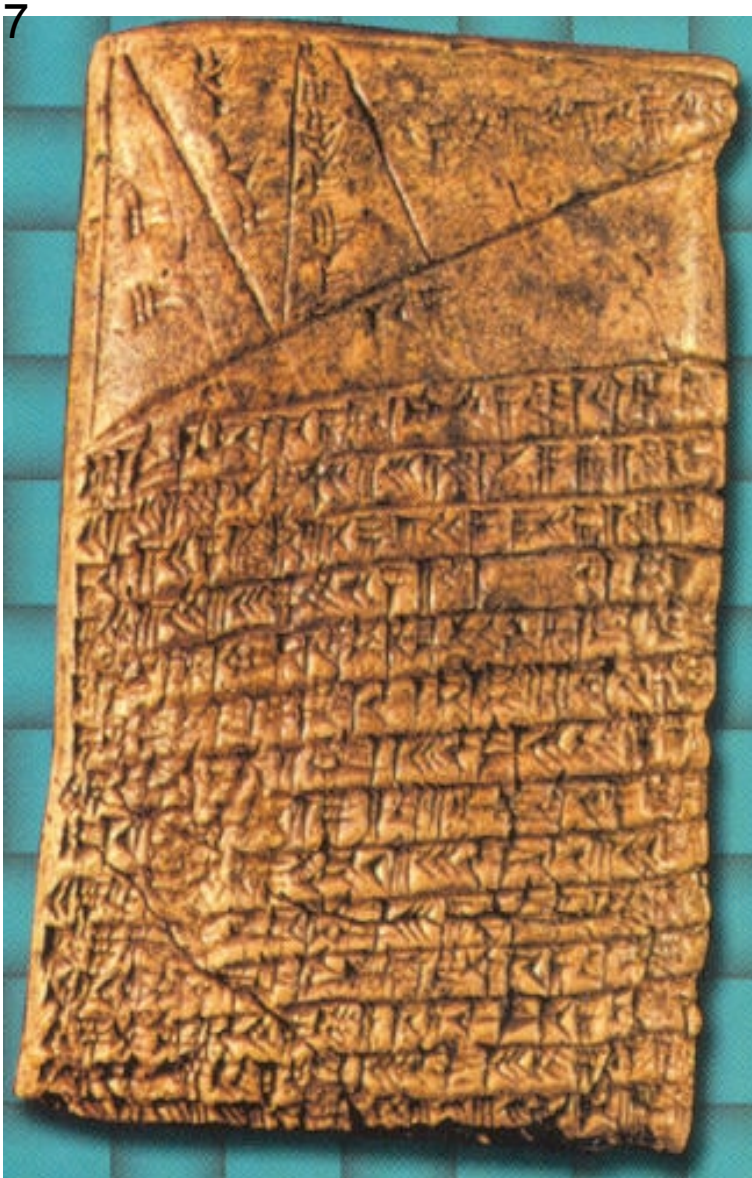
# PROPORZIONI E SIMILITUDINE



# PROPORZIONI E SIMILITUDINE

## TRIANGOLI SIMILI

IM5535      Datazione: 1800 aC circa





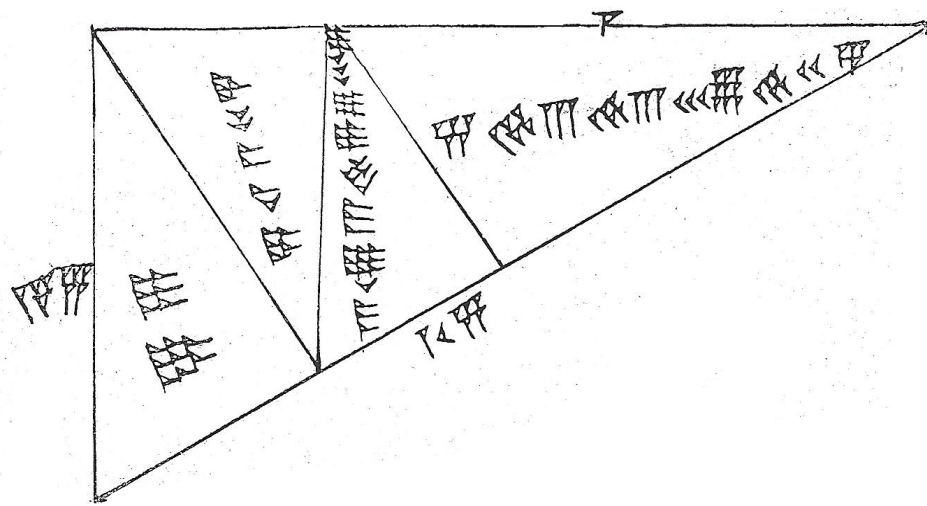
# TRIANGOLI SIMILI

1-4 dati

5 domanda

6-16 risposte con calcoli

The image shows a handwritten mathematical solution on a grid. At the top, a large right-angled triangle is drawn, divided into three smaller triangles by a vertical line from the top vertex to the hypotenuse. The triangles are labeled with numbers 1, 2, 3, and 4. Below the diagram, the text '1-4 dati' is written. The next section, labeled '5 domanda', contains a series of calculations and diagrams. The text '6-16 risposte con calcoli' is written below this. The bottom of the page features the word 'Edge' and the number '10'. At the very bottom, the number 'IM.55357' is written.



1-4 dati

$$AC = [1, ] = 60,$$

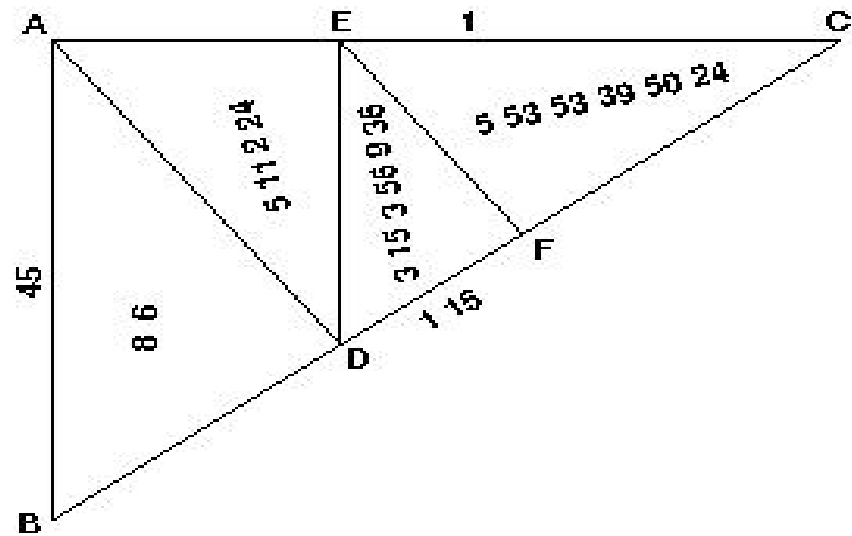
$$BC = [1, 15] = 75,$$

$$AB = [45] = 45$$

NOTA:

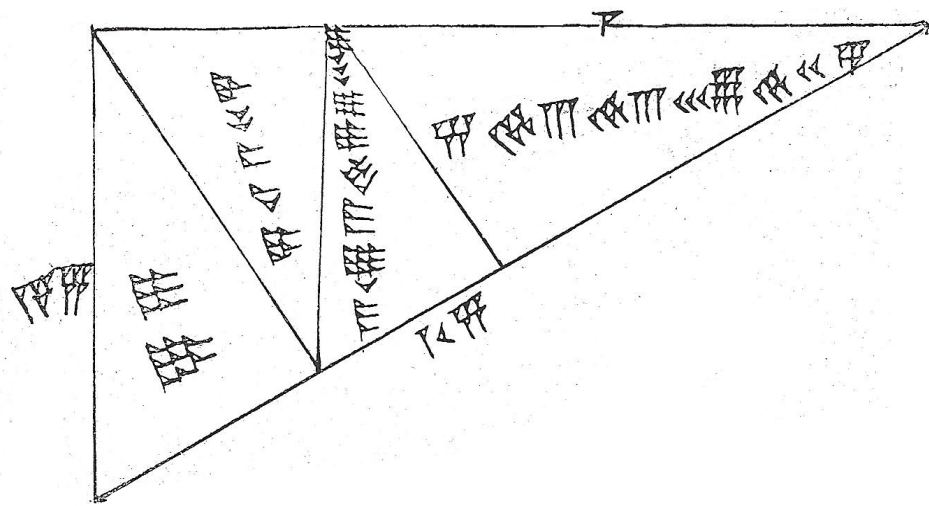
Il triangolo è rettangolo

3, 4, 5      terna pitagorica  
 $45 = 3 \times 15$ ,  $60 = 4 \times 15$ ,  $75 = 5 \times 15$

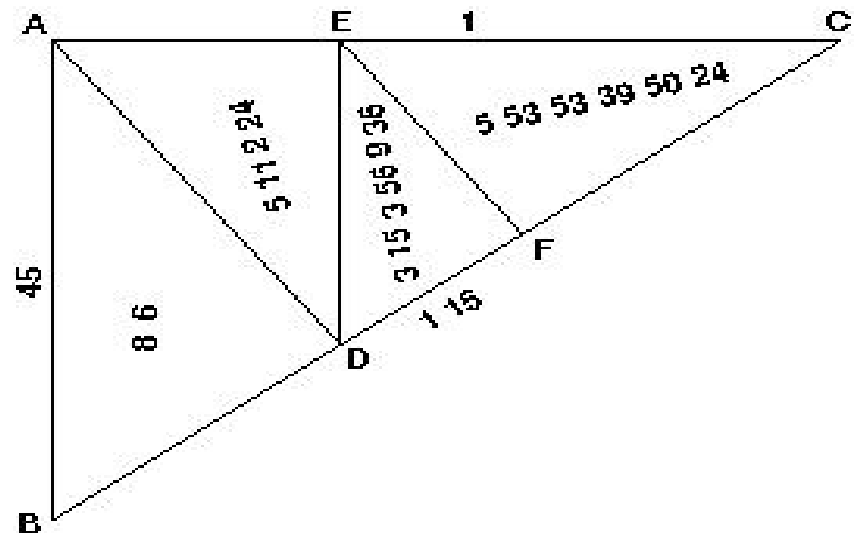


Un cuneo (triangolo). La lunghezza è 1, la lunghezza lunga è 1 25, la larghezza di sopra è 45.

L'area completa è 22 30,  
 l'area più in alto 8 6,  
 quella successiva 5 11; 2 24,  
 la terza 3 19; 3 59, 9, 36  
 quella più in basso 5,53 ;53,39,50,24



1-4 dati



$$AC = [1, ] = 60, \quad = 1$$

$$BC = [1, 15] = 75, \quad = 1.25$$

$$AB = [45] = 45 \quad = 0.75$$

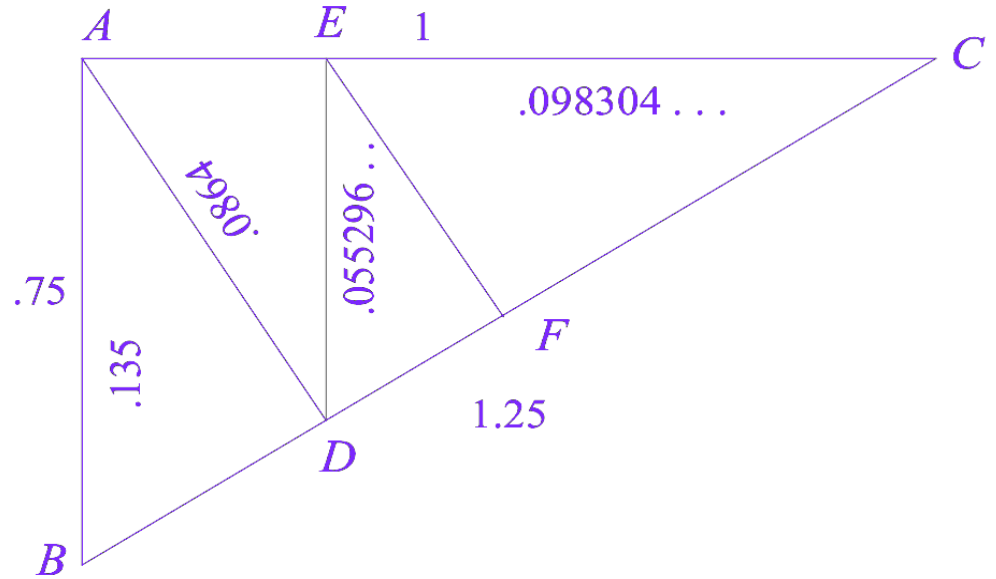
$$ABC = [22, 30] = 1350 \quad = 0.375$$

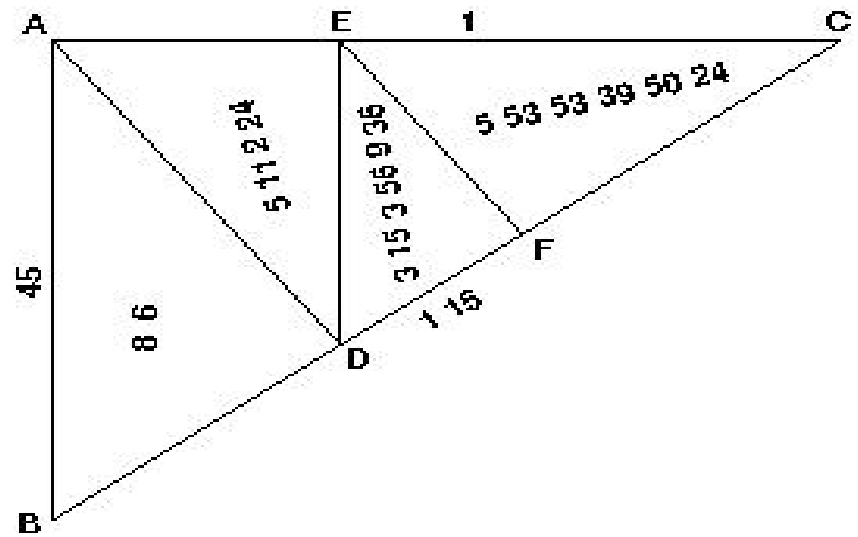
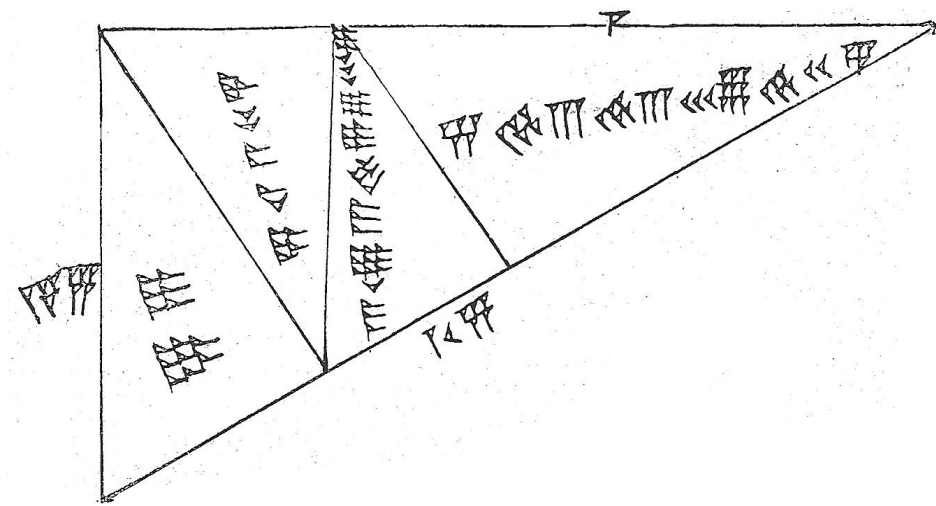
$$ABD = [8, 6] = 486 \quad = 0.135$$

$$EAD = [5, 11 ; 2, 24] = 311.04, \quad = 0.0864$$

$$FDE = [3, 19 ; 3, 59, 9, 36] = 199.0664333 \quad = 0.055296$$

$$FEC = [5, 53 ; 53, 39, 50, 24] = 653.8944 \quad = 0.098304$$



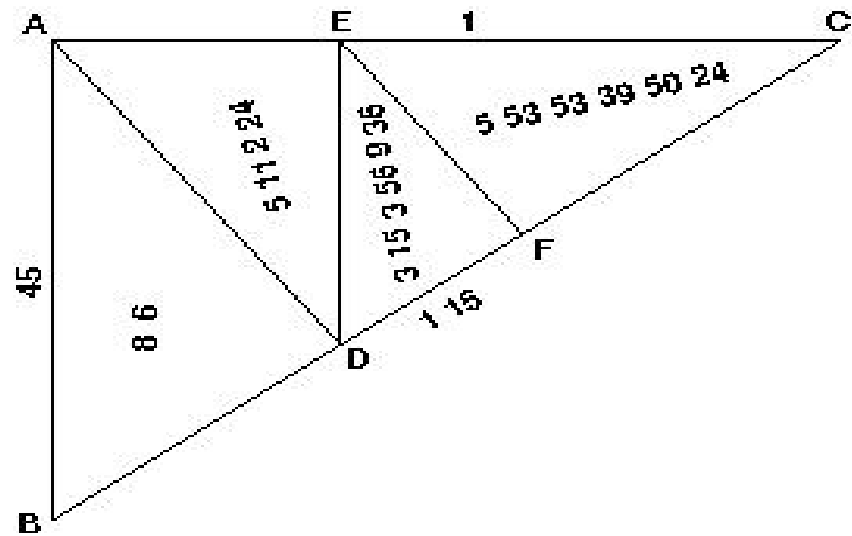
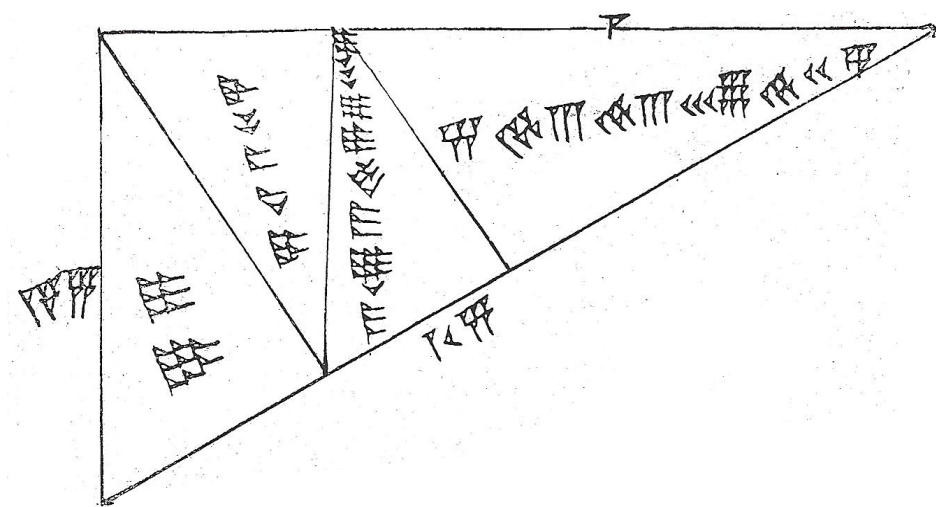


5. domanda

*tenendo conto del testo che segue  
(lo scriba non finisce)*

BD,  
AD,  
AE,  
ED.

Quali sono  
la lunghezza superiore  
la lunghezza del segmento ("spalla")  
la lunghezza in basso  
la perpendicolare?



anche  $BAD$ ,  $ADE$ ,  $DEF$ ,  $EFG$   
rettangoli ?

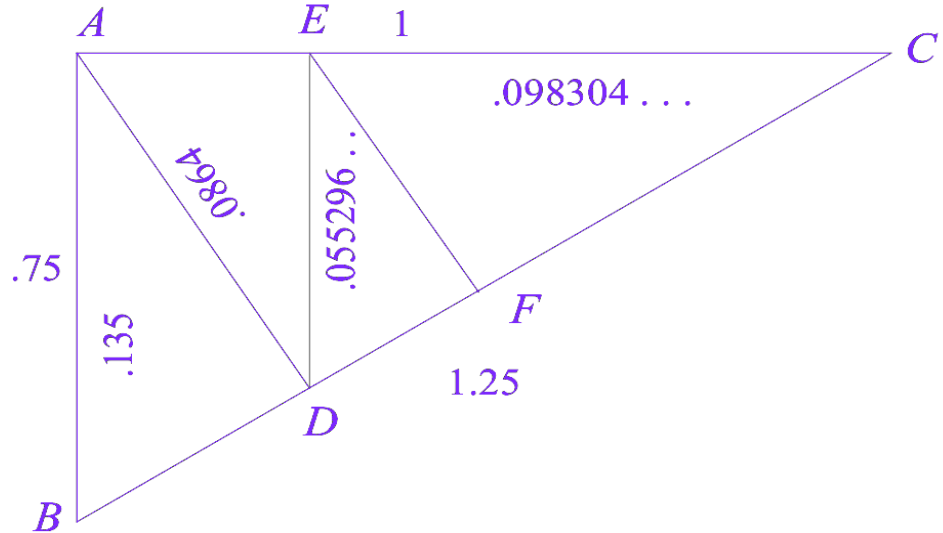
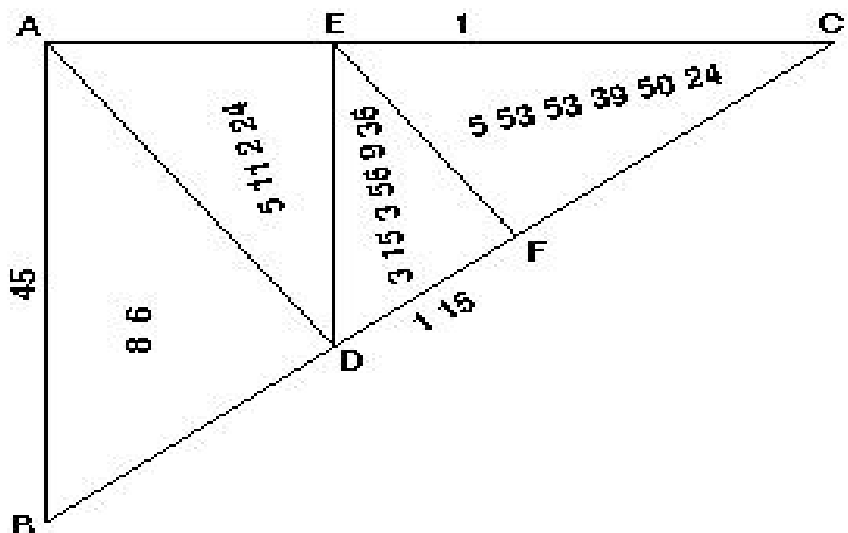
dal disegno e dall'uso  
dal procedimento  
dai dati

$BD$ ,  
 $AD$ ,  
 $AE$ ,  
 $ED$ .

Si usa la "similitudine" dei triangoli  
 $ABC$ ,  $DAB$ ,  $EAD$ ?

I babilonesi conoscevano la "similitudine"?

Quali sono  
la lunghezza superiore  
la lunghezza del segmento ("spalla")  
la lunghezza in basso  
la perpendicolare?



6-16. soluzione BD

$\Delta ABC \sim \Delta DBA$

triangolo ABC simile al triangolo DBA

$$\frac{AB}{AC} = \frac{BD}{AD}$$

$$\frac{45}{60} = \frac{3}{4} = \frac{BD}{AD}$$

$$AD = \frac{4}{3} BD$$

$$a(ABD) = \frac{1}{2} (BD)(AD)$$

$$a(ABD) = \frac{1}{2} \frac{4}{3} BD^2$$

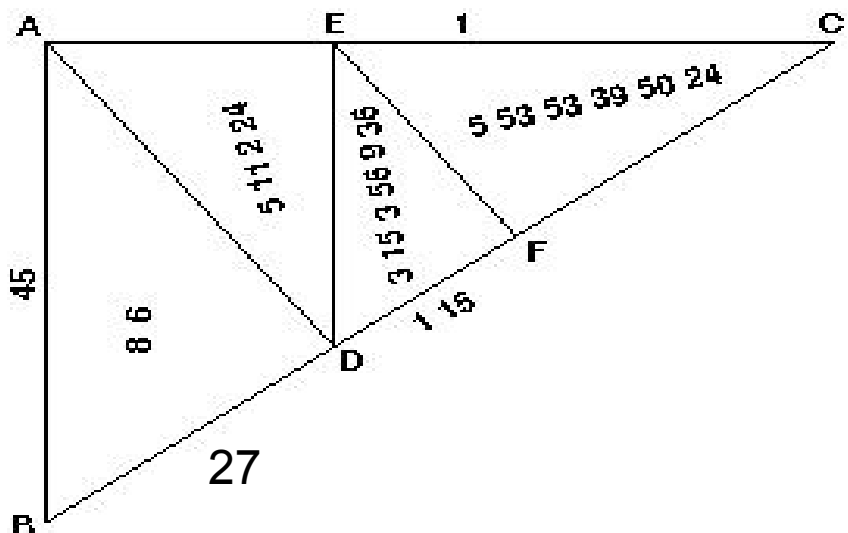
$$BD^2 = 2 \frac{3}{4} a(ABD) \quad [;8,6]$$

$$BD^2 = 2 \frac{3}{4} \cdot 135 \quad [;8,6]$$

[;0,27]

↑  
triangolo rettangolo

→  $BD = .45$



6-16. soluzione BD

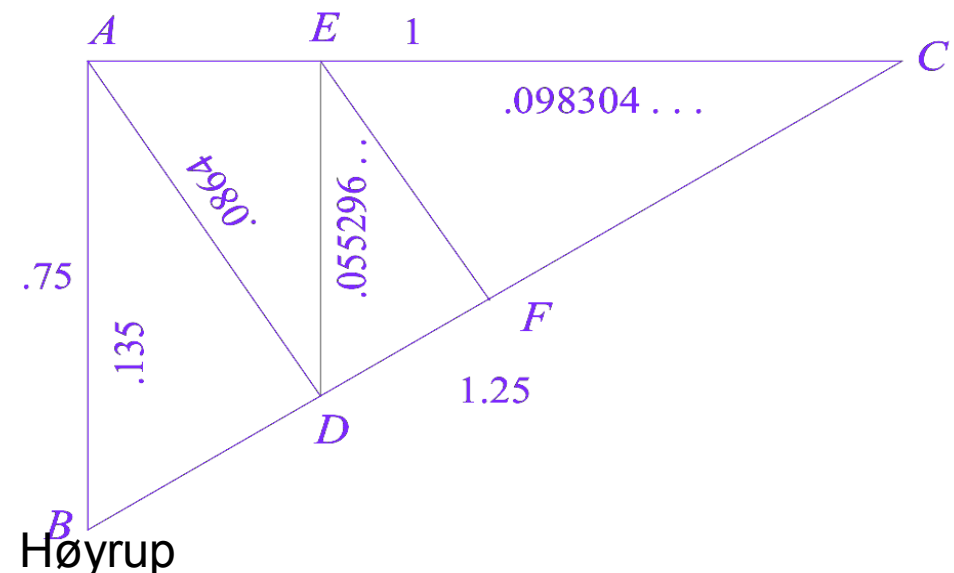
0;45

$$\frac{45}{60} = \frac{3}{4} \rightarrow \frac{AB}{AC} = \frac{BD}{AD} \quad ?$$

0;45 [;8,6]

$$2\frac{3}{4} a(ABD) = BD^2$$

$$BD = \sqrt{2\frac{3}{4} a(ABD)} = \sqrt{0;12} = 0;2\frac{7}{7} = \sqrt{.2025} = 0.45$$



Tu, per sapere come si procede, igi 1, la lunghezza, per 0;45 eleva

“igi” = “l'inverso di”

l'inverso di AC per AB

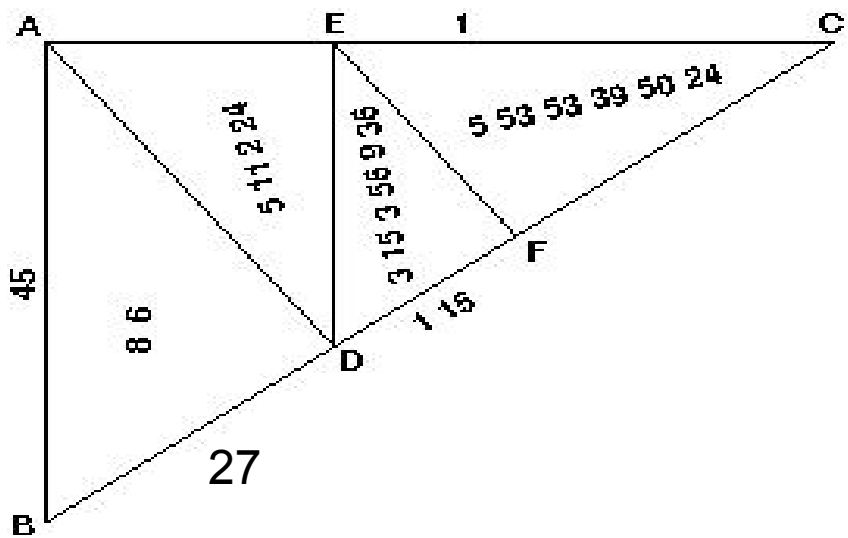
$$\frac{3}{4} = [0,45]$$

0;45 vedi

0;45 per 2 moltiplica  
1;30 vedi  
per 0;08 06, la superficie più in alto, moltiplica

0;12 9 vedi. Per 0;12 9, qual è il lato del quadrato?  
0;27 è il lato del quadrato





6-16. soluzione

$AD$

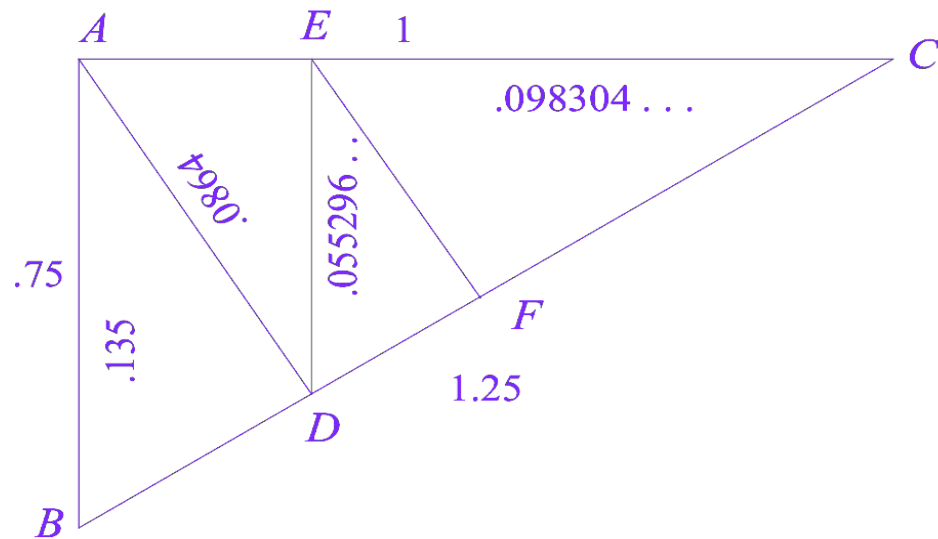
$$\frac{3}{4} = \frac{BD}{AD}$$

$$AD = \frac{4}{3} BD$$

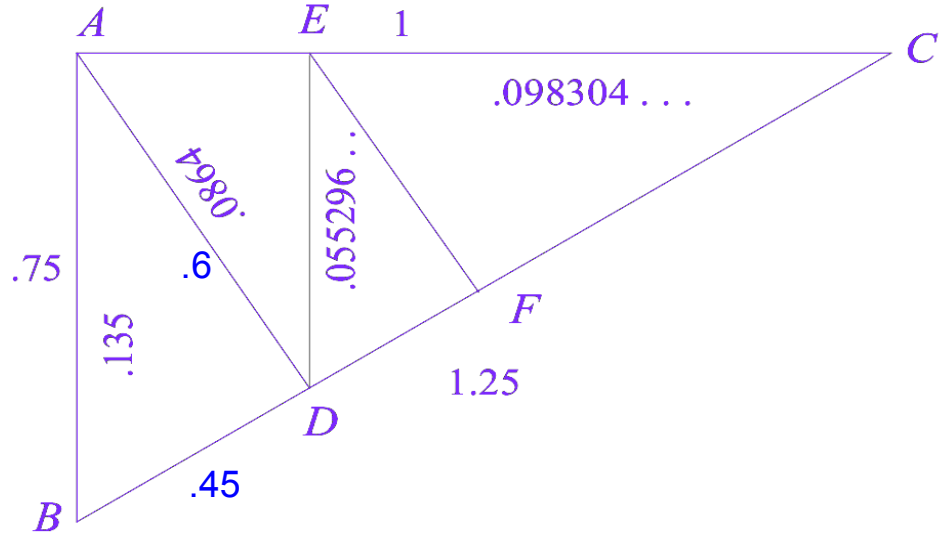
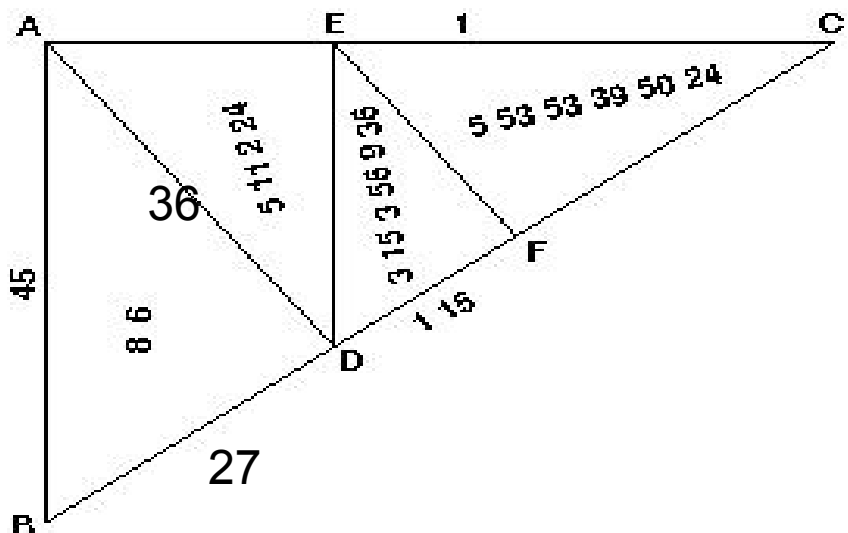
oppure

$$a(ABD) = \frac{1}{2} (BD)(AD)$$

$$AD = 2 \frac{a(ABD)}{BD}$$







6-16. soluzione AD

$$\frac{BD}{2} \rightarrow \frac{2}{BD} : \frac{[0;27]}{2} = 13;3$$

$$\frac{.45}{2} = .225 \rightarrow \frac{1}{.225}$$

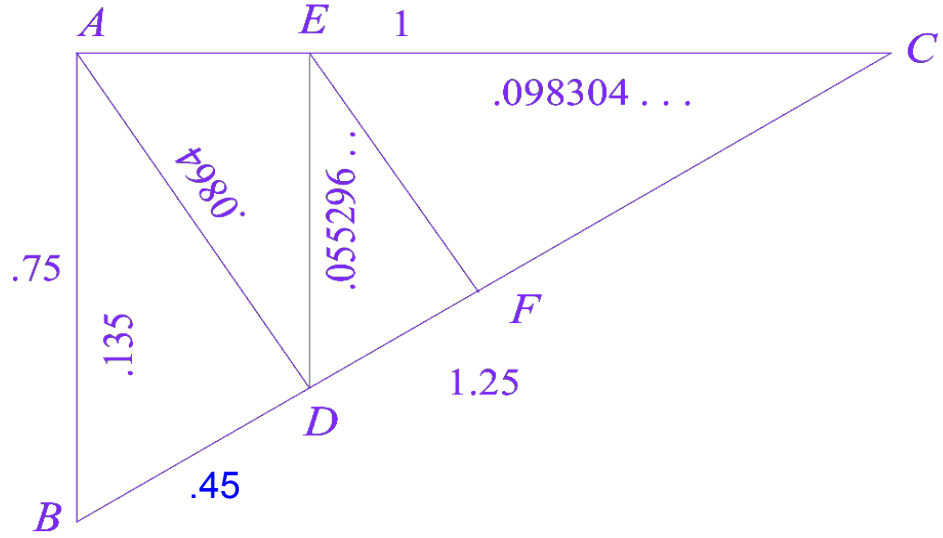
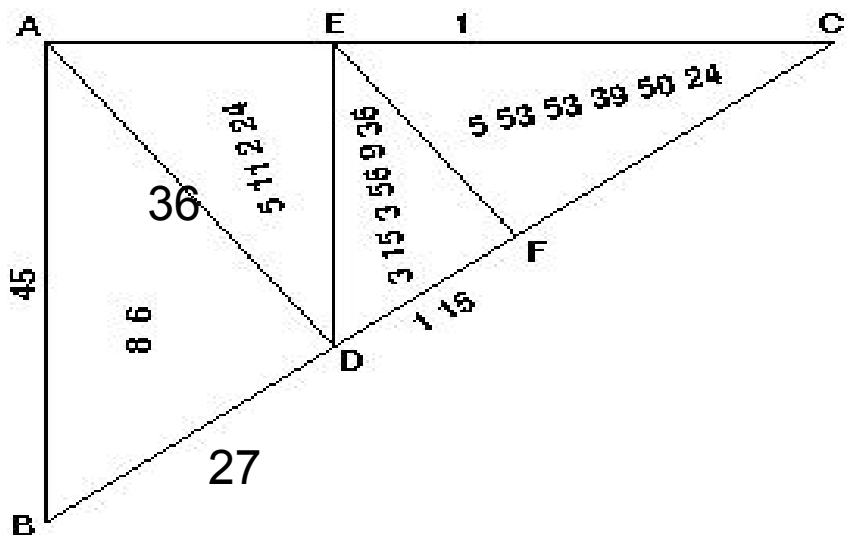
$$\frac{2}{BD} a(ABD) = AD \left( \frac{1}{.225} \right) (.135) = .6$$

[0;36]

$$a(ABD) = \frac{1}{2} (BD)(AD)$$

$$AD = 2 \frac{a(ABD)}{BD}$$

Dividi [in due] 0;27.  
 Vedrai 0;13 30.  
 Igi di 13;30. reciproco  
 Moltiplica per 0;08 06, l'area di sopra  
 Vedi 0;36, la lunghezza che è la  
 corrispondente di 0;45, la larghezza.



6-16. soluzione

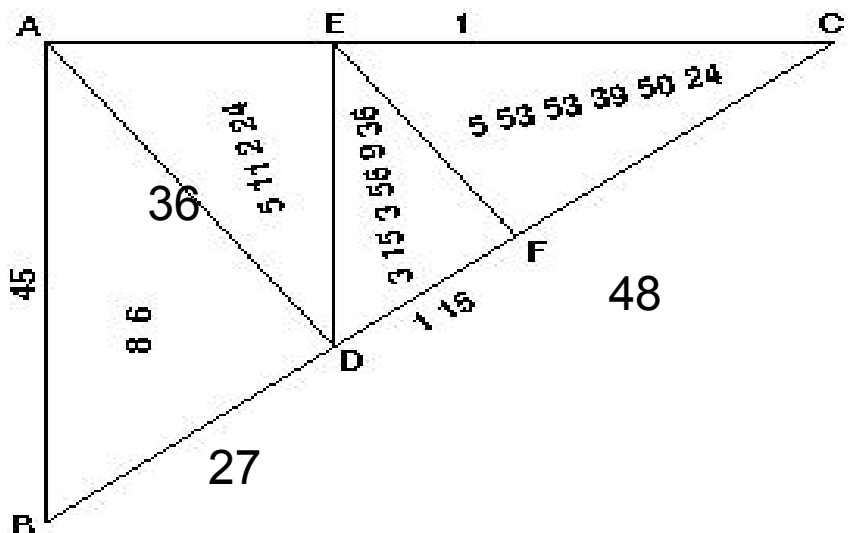
In un triangolo rettangolo 3 4 5

$$a(ABC) = \frac{1}{2} \cdot \frac{3}{4} a^2$$

in un triangolo rettangolo r s t

$$a(ABC) = \frac{1}{2} \cdot \frac{r}{s} a^2$$

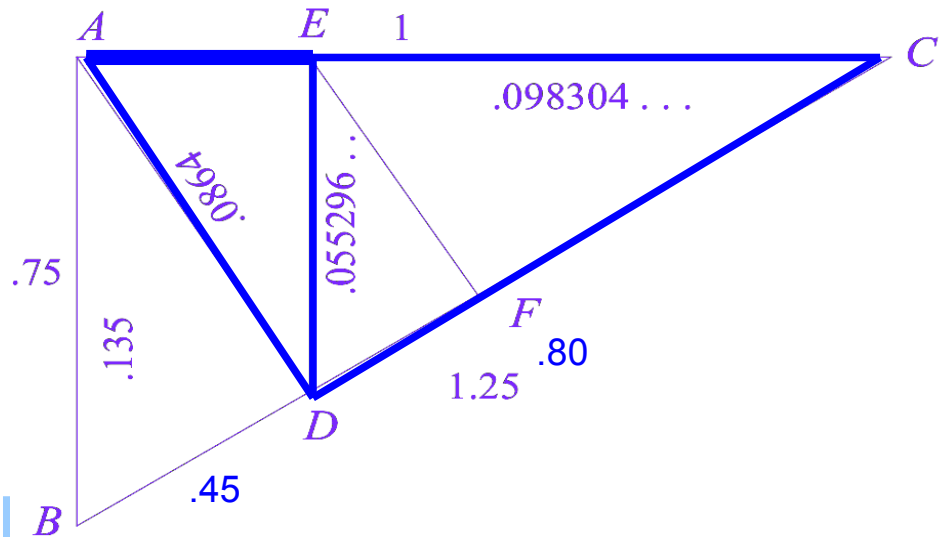




6-16. soluzione *ED*

Dividi in due 0;21 36  
 Vedi 0;10 48  
 Igi di 0;10 48

.....  
*Il testo è interrotto*



*AD*

Dividi [in due] 0;27.  
 Vedi 0;13 30.  
 Igi di 13;30. reciproco  
 Moltiplica per 0;08 06, l'area di sopra  
 Vedi 0;36, la lunghezza che è la  
 corrispondente di 0;45, la larghezza.

