HELPING NATURE

Archimedes' machines explained by Galileo.

An exhibition of

IL GIARDINO DI ARCHIMEDE
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Over two thousand years ago, Archimedes studied and put into function a series of machines and devices, some of which still bear his name, and that for centuries formed the foundation of mechanics.

Archimedes machines are based on two cornerstones: the simplicity of their design and the mathematical structure of the theory. Two complementary principles, since it would not be possible to find mathematical rules of complex machines nor one could give precise laws outside the language of mathematics. But the simplicity and clarity of mathematics also produce another effect: permanence. Permanence as objects, because they are not subject to breakage and wear; permanence as mechanisms, because their very simplicity makes them difficult to improve and therefore almost eternal.

Almost twenty centuries later, the same machines are studied and described by Galileo, who outlines a theory of such beauty and simplicity that can still be taken as the basis of today's mechanics and constitutes a dialogue between the highest mathematician of antiquity and the founder of modern science.

1. What's the use of machines?

In a machine, the acting force is to the resistence as the displacement of the resistence to that of the force:

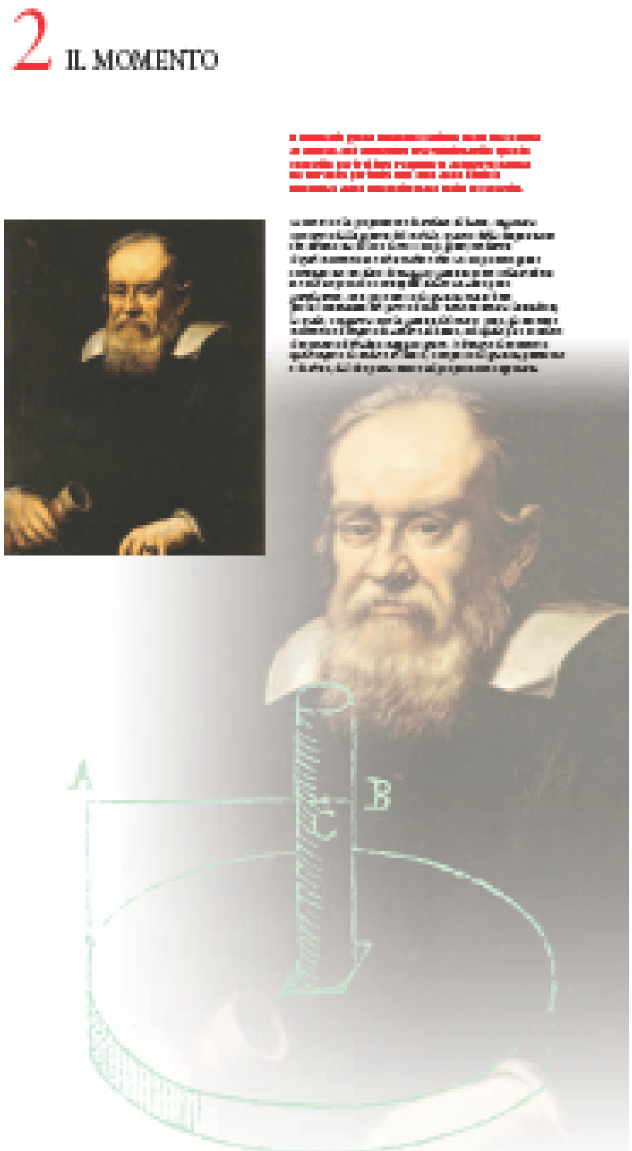
$F : R = S_R : S_F$

Taking our start, then, from this consideration, there lie before us at first four things to be considered; the first is the weight to be transferred from one place to another; second is the force or power that must move it; third is the distance between the beginning and the end of the motion; and fourth is the time in which the change must be made—which time comes to the same thing as the swiftness and speed of the motion, that motion being determined to be speedier than another which passes an equal distance in less time. Now assigning any determined resistance, and delimiting any force, and noting any distance, there is no doubt whatever that the given weight will be conducted by the given force to the given distance; for even though the force be very small, by dividing the weight into many particles of which each shall not remain superior to the force, and transferring them one at a time, the whole weight will finally be conducted to the appointed place.

But since it may sometimes happen that, having but a small force, we need to move a great weight all at once without dividing it into pieces, on such an occasion it will be necessary to have recourse to the machine, by means of which the given weight will be transferred through the assigned space by the given force; yet this does not remove the necessity for that same force to travel and measure that same (or an equal) space as many times as it is exceeded by the said weight. So that at the end of the action we will find that the only profit we have gained from the machine is to have transported the given weight in one piece with the given force to the given end; which weight, divided into pieces, would have been transported without any machine by the same force in the same time through the same distance.

2. The moment.

The notion of moment plays a central role in Galileo's mechanics. Maurolico had already introduced this concept to treat the equilibrium of heavy bodies; Galileo makes it the backbone not only of statics, but also of the kinematics of accelerated motion.

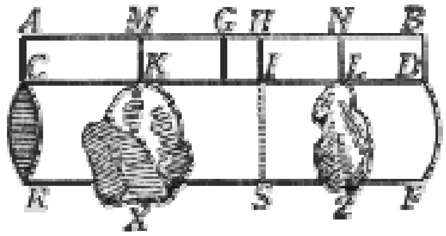


Moment is the tendency to move downward caused not so much by the heaviness of the movable body as by the arrangement which different heavy bodies have among themselves. It is through such *moment* that a less heavy body will often be seen to counterbalance some other of greater heaviness, as in the steelyard a little counterweight is seen to raise a very heavy weight, not by excess of heaviness, but rather by its distance from the suspension of the steelyard. This, combined with the heaviness of the lesser weight, increases its *moment* and impetus to go downward, with which it may exceed the *moment* of the other, heavier weight. Thus *moment* is that impetus to go downward composed of heaviness, position, and of anything else by which this tendency may be caused.

3. The proof of the law of the lever.

The first mathematical proof of the law of equilibrium is found in Archimedes, who had even given two different demonstrations. Galileo takes over the second of these, simplifying and generalizing it.

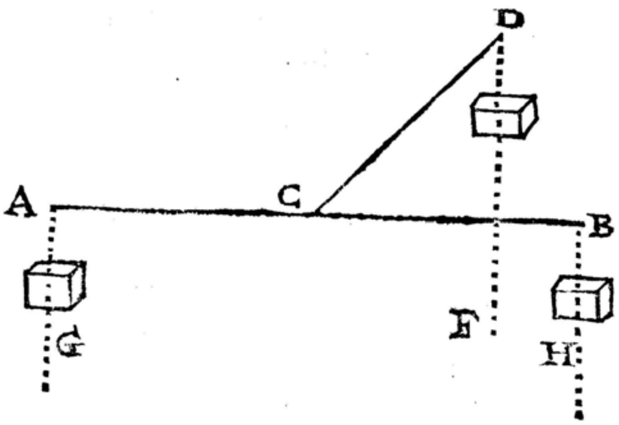
Imagine the heavy solid CFDE, of uniform density and of uniform size throughout, such as a cylinder or similar figure. Let this be suspended by its endpoints C and D, from the line AB, equal in length to the solid. Now dividing this line AB equally at the point G, and suspending it from this point, there can be no doubt that it will balance in this point G, ...



Next suppose the said heavy body to be cut into two unequal parts along the line IS... suppose a new string to be added to the point I, ... this then sustains jointly both parts of the solid in their pristine state. Hence it follows that no change being made either of weight or of position in the parts of the solid with respect to the line AB, the same point G remains the center of equilibrium as it has been from the first. Moreover, since the part CS of the solid is connected to the balance through the two strings CA and IH, there can be no doubt that if we cut these two strings and add a single other at MK, equidistant from these two, then since the center of gravity of the solid CS lies directly beneath this, the solid will not change or move its place .. and the same with the other part IF ... Hence the parts of the whole solid CF being the same with respect to the balance AB as they have been all along, there is no doubt that equilibrium will still exist at the same point G. Now ... the line MH being one-half the line HA, and NH being half of HB, all MN will be one-half the whole line AB, of which BG is also one-half. Hence MN and GB will be equal to one another; and from these taking away common part GN, the remainder MG will be equal to the remainder NB, to which NH is likewise equal; whence MG is equal to NH; and adding the part GH to both, MH will equal GN. Now, having already demonstrated that MG equals HN, that proportion which the line MH has to HN, the distance NG will have to the distance GM; but the proportion MH to HN is that of KJ to JL, and of its double CJ to the double JD—and in a word, of the solid CS to the solid SD, of which solids the lines CI and ID are the lengths. Hence it is concluded that the ratio of the distance NG to the distance GM is the same as that of the size of the solid CS to the size of the solid SD; which, manifestly, is the same as the ratio of the weights of those same solids.

4. The equilibrium of the angular lever.

In evaluating moment and equilibrium, the distance should not be taken from the fulcrum to the weight, but from the fulcrum to the vertical line passing through the weight.

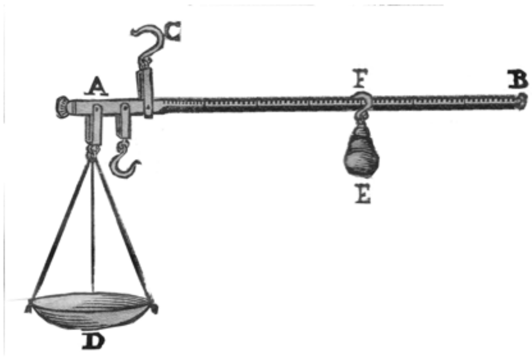


There is another thing that must be considered before proceeding further, and this concerns the distances at which heavy bodies come to be weighed; for it is very important to know the sense in which equal and unequal distances are to be understood, and in what manner they must be measured. For given the straight line AB and two equal weights hanging from its extremities, and the point C being taken in the middle of this line, then equilibrium will exist at this point, and this is so because the distance AC equals the distance CB. But if the line CB is elevated and turned about the point C, it will be transferred to CD, so that the balance stands according to the lines AC and CD, and then the two equal weights hanging from the ends A and D will no longer weigh equally upon the point C, because the distance of the weight placed at D is made less than it was when located at B.

5. The steelyard.

The first application of the law of equilibrium takes place in the steelyard: the weight of the goods is to the weight of the *romano* as the distance of the *romano* is to the distance of the goods:

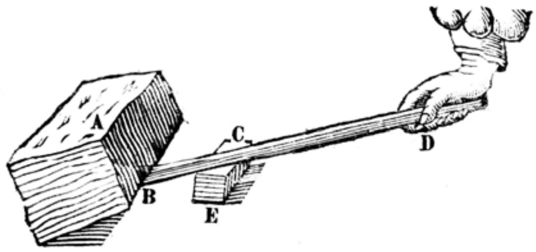
$$D:E = CF: CA.$$

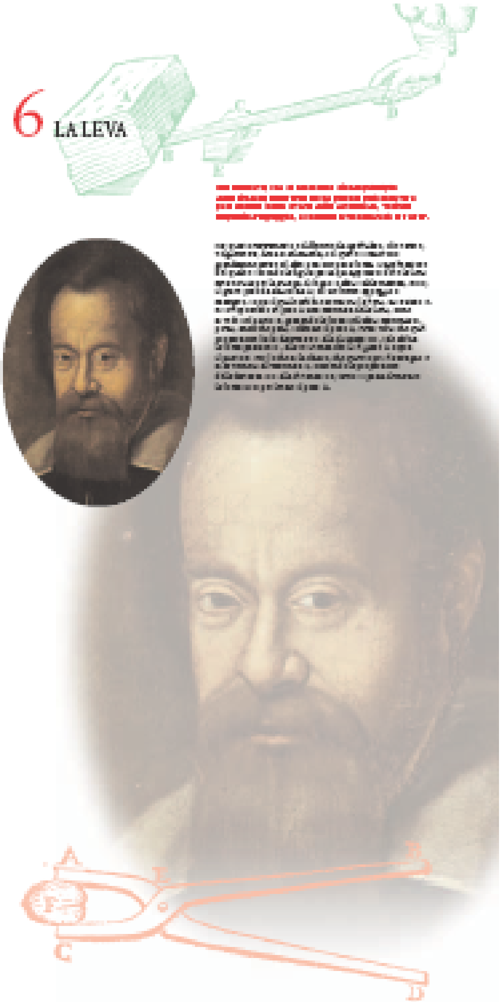


And first, speaking of the steelyard, a very widely used instrument with which various kinds of merchandise are weighed, even though extremely heavy, by the weight of a small counterpoise (commonly called the *romano),* we shall prove that in such operations nothing more is done than to reduce to a practical act precisely that on which we have theorized above. For let us suppose *AB* to be a steelyard whose support (called *trutina)* is at the point *C,* near which at a small distance *CA* hangs the heavy weight *D;* and along the greater distance *CB* (called the *ago* of the steelyard) there runs back and forth the *romano E,* of small weight in comparison with the heavy body *D,* yet nevertheless capable of moving far enough from the *trutina C* so that the proportion existing between the two weights *D* and *E* may exist between the distances *FC* and *CA;* and then equilibrium will be made, unequal weights being found hanging at distances inversely proportional to them.

6. The lever.

With a lever, which is based on the same principle of the steelyard, even a small force can lift heavy weights. As Archimedes said, "Give me a fulcrum and with a lever I will move the Earth".

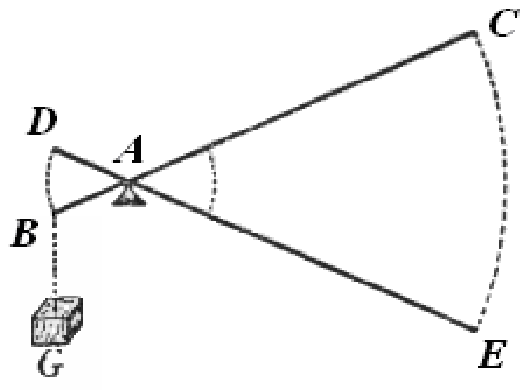




Nor is this instrument different from that other called the *vette,* or commonly the lever, with which very large stones and other weights are moved with a small force. Its application is according to the next diagram where the lever is denoted by the bar *BCD,* of wood or other solid material; the heavy weight to be raised is *A,* and a firm support or fulcrum upon which the lever presses and moves is designated *E.* Placing one end of the lever under the weight *A,* as is seen at the point *B,* the force weighing down at the other end *D,* though small, will be able to raise the weight *A,* so long as the proportion of the distance *BC* to *CD* exists between the force placed at *D* and the resistance made by the heavy body *A* upon the point *B.* From which it is made clear that the more closely the support *E* approaches the extremity *B,* increasing the proportion of the distance *DC* to the distance *CB,* the more the force at *D* may be diminished in raising the weight *A.*

7. The Capstan 1.

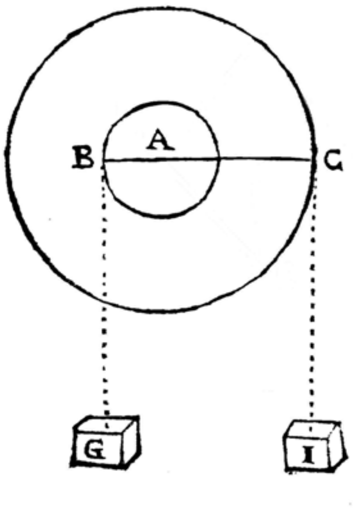
While lifting a weight, the lever moves and its action becomes uncomfortable. In the capstan, which obeys the same principle, the force is always applied at the same point.

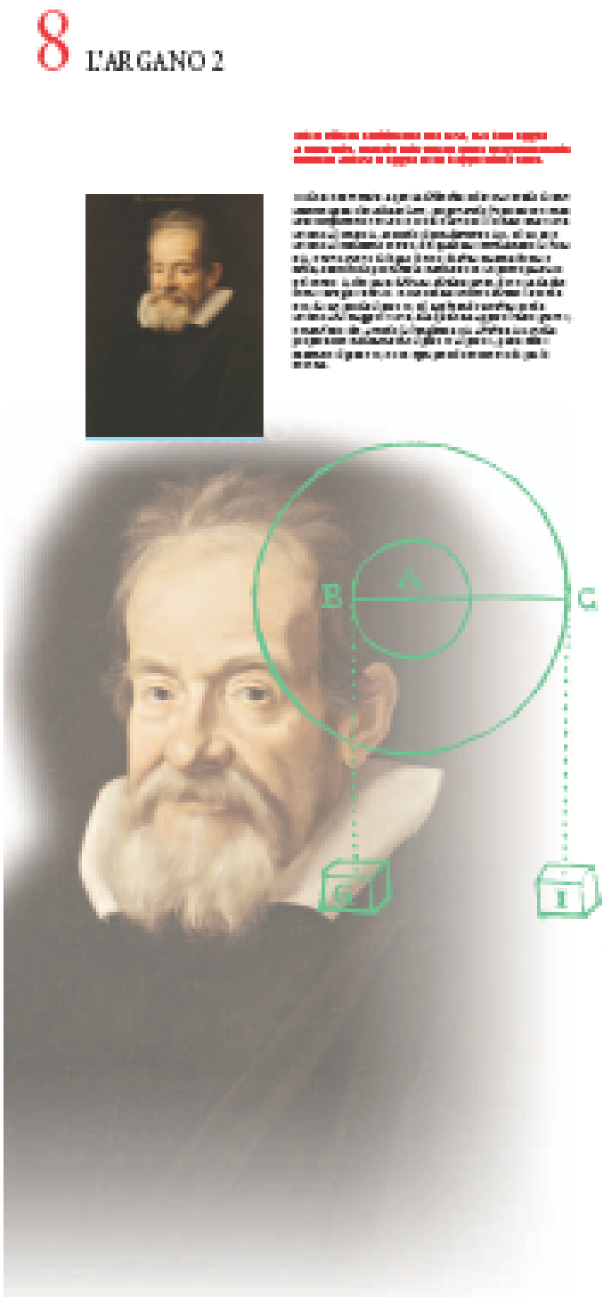


The two instruments whose nature we are now about to explain depend directly upon the lever, and indeed are nothing but a perpetual lever. For if we think of the lever BAC supported at the point A, and the weight G hanging from the point B, the force being placed at C, it is evident that by transferring the lever to the position DAE, the weight G will rise through the distance BD, but that it cannot continue to be elevated much more. Accordingly if it is still desired to raise it further, it will be necessary to fix it in this position with some other support, and return the lever to its previous place BAC; then, taking hold of the weight again, to raise it once more through a similar height BD. In this way, doing the same thing many times, the raising of the weight may be accomplished with an interrupted motion, which in many respects may turn out to be not very convenient.

8. The Capstan 2.

This is achieved by replacing the lever, which is a radius of a wheel, by the full wheel in which at any time the radius acts as a lever to which force is applied.

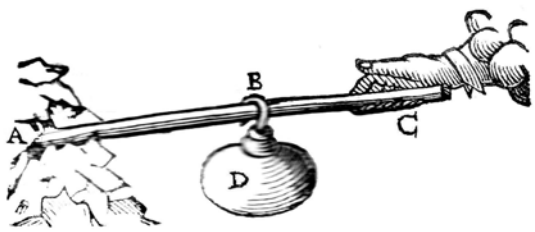




Hence this difficulty has been overcome by finding a way of uniting together as it were infinite levers, perpetuating the operation without any interruption whatever. This is done by making a wheel of radius AC about the center A, with an axle of radius AB, all of stout wood or some other firm and solid material, and then sustaining the entire framework on a pivot at the center A which passes from one side to the other and is mounted on two strong supports. The cord DBG, from which hangs the weight G, passes round the axle, and another cord is applied round the larger circle, from which is hung the weight I. And the length CA being to AB in the same ratio as the weight G to the weight I, the latter will be able to sustain the weight G, and with any little additional moment will move it.

9. Different kind of lever.

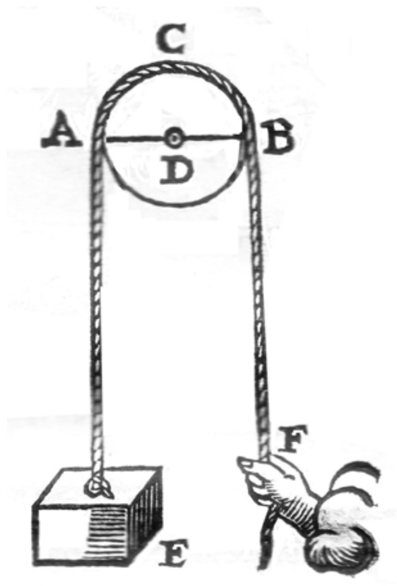
In the normal lever (of the first kind) the fulcrum lies between the force and the weight. However, we can arrange things differently; in the lever of the second kind the weight is in the middle, in that of the third kind the force is at the center. In the latter case, the force is always greater than the weight; in the lever of the second kind it is always less; in that of the first kind is greater if the fulcrum is closer to the force, it is less if it is closer to the resistance.

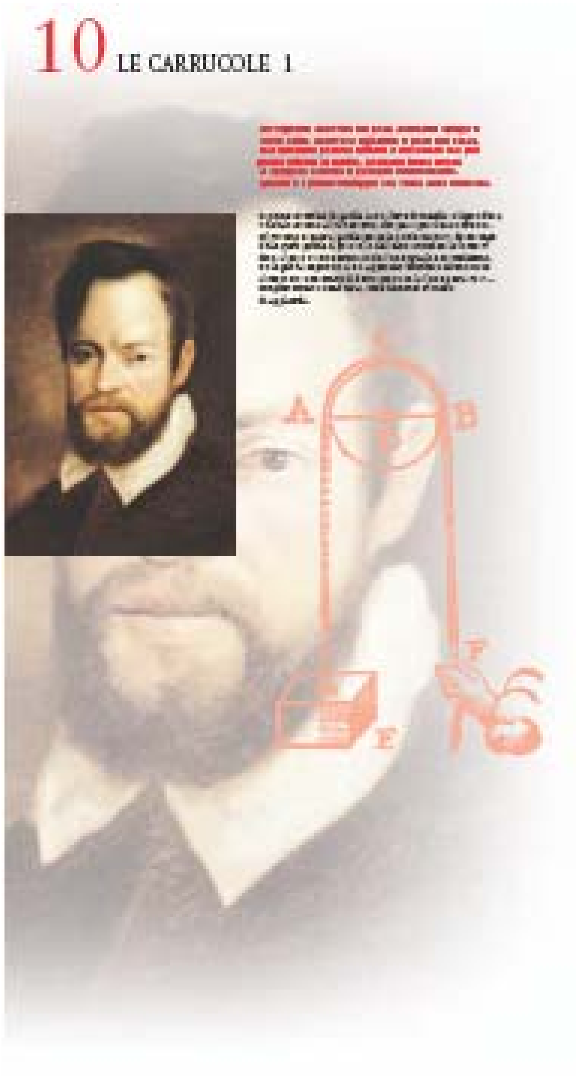


The use of the lever explained above placed the weight at one of its extremities and the force at the other, while the support was placed somewhere between the extremities. But we can use the lever in another way too, placing the support at the extremity A (as seen in the diagram), the force at the other extremity C, and hanging the weight D from some point between, as at the point B.

10. The pulleys 1.

If we want to lift a weight, we must push it upward. But if we tie a rope to the weight, and we place round a circle that can rotate around its center, we can push rather than pull, thus lifting the weight more comfortably. This is the first advantage that comes from the pulleys.



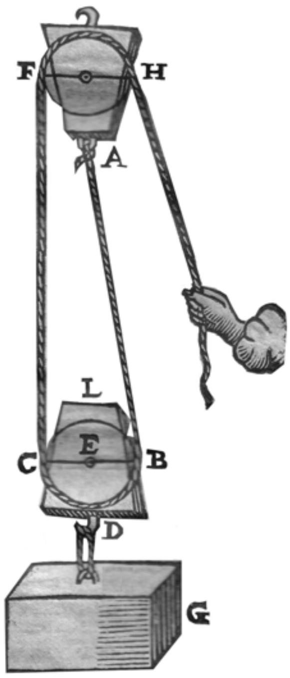


And first think of the pulley ABC made of metal or hard wood, turning about its axle that passes through the center D; and round this pulley place the cord EABCF, from one end of which hangs the weight E, and at the other assume the force F. I say that the weight will be sustained by a force equal to itself, nor will the upper pulley ABC give any benefit with regard to moving or sustaining the said weight with the force placed at F ...considered simply, but only in the mode of applying it.

11. The pulleys 2.

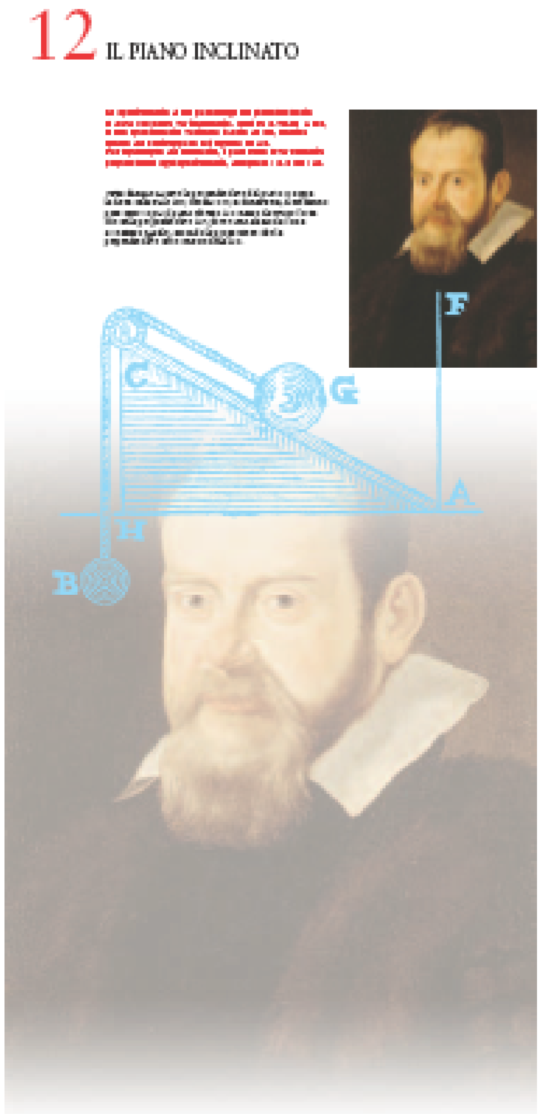
But if we fix one end of the rope and attach the weight to the pulley, it will be sufficient to pull the other end with a force equal to half the weight. If you then want to pull down more comfortably, just put another pulley. With a systems of many pulley we can decrease further the necessary force.

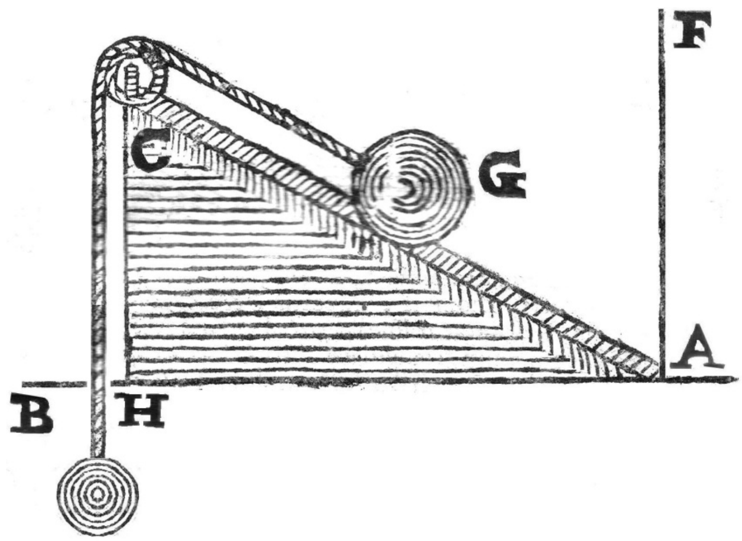
But if we make use of a similar machine in another manner, as we are now about to explain, we can lift the weight with less force. For let there be a pulley BDC turning about the center E, arranged in its box or housing L, from which is suspended the heavy body G; and pass about the pulley the cord ABDCF whose end A is fixed to some stable fastening. Let the force be placed at the other end F, which, moving toward H, will raise the framework BLC and accordingly the weight G; and in this operation I say that the force at F will be one-half the weight sustained by it. … Next, consider that the force placed at *F* must move upward in order to raise the weight … To overcome this inconvenience, a remedy has been found by adding another pulley above, as seen in the next diagram.



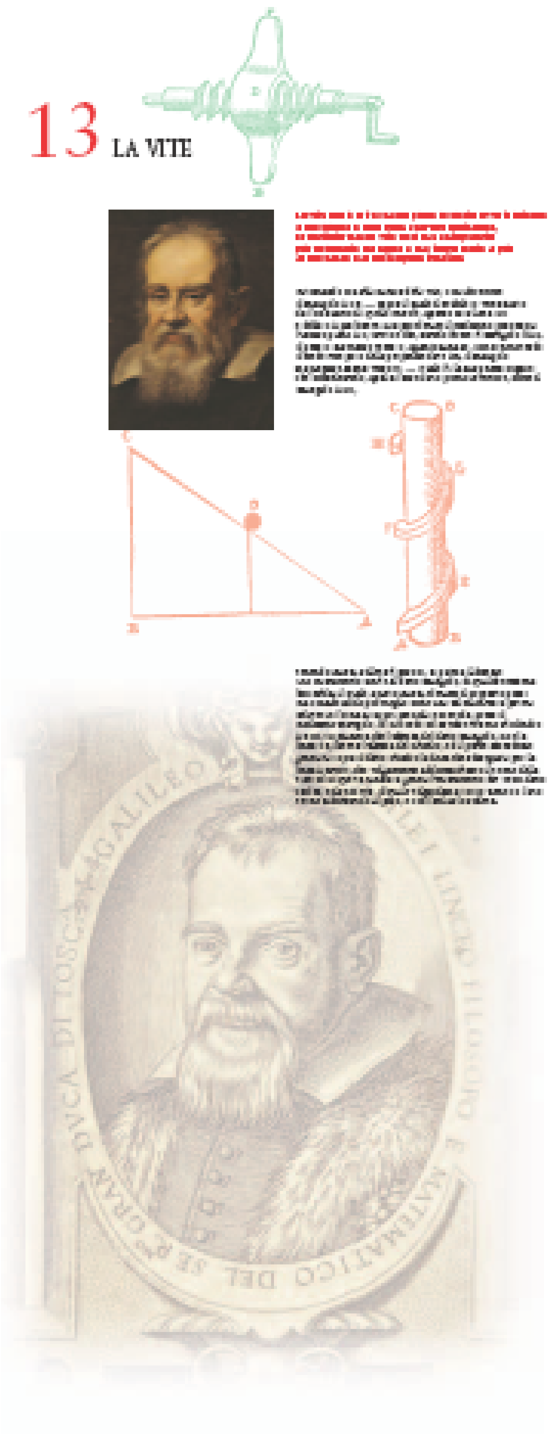
12. The inclined plane.

The displacement of a weight along an inclined plane must be calculated vertically. So if G goes from A to C, its vertical displacement is given by CH, while that of the counterweight B is equal to AC. For the principle of the moment, the weights are inversely proportional to their displacement, then B: G = CH: AC.



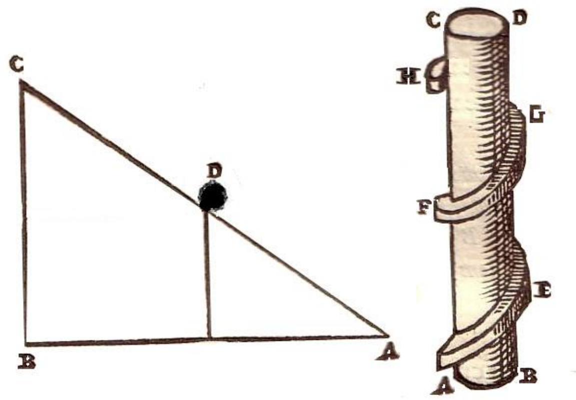


From the point C, let fall the perpendicular CH upon the horizontal line AR. It will be demonstrated that the same weight will be moved upon the inclined plane AC by less force than in the perpendicular AF (where it will be raised by a force equal to itself), in proportion as the perpendicular CH is less than AC.



13. The screw.

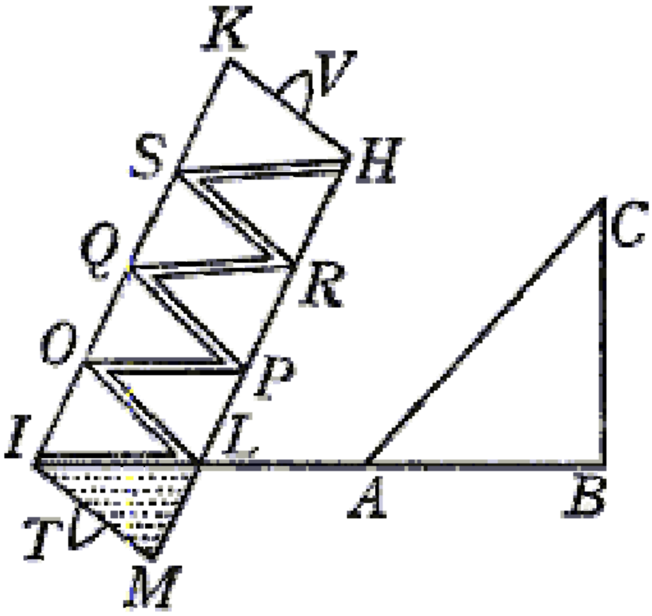
The screw is simply an inclined plane wrapped around a cylinder or a cone. In everyday use, the screw thread causes it to penetrate more easily into the wood and to hold much more than a nail or a simple piece.



Returning now to our original purpose, which was to investigate the nature of the screw, let us consider the triangle ACB, ... upon which the movable body D will be drawn by a force as much less than itself as the line BC is shorter than CA. Now to raise the same weight on the same plane AC with the triangle CAB standing still and the weight D being moved toward C, is the same thing as if the weight D were not moved from the perpendicular DJ while the triangle was being driven forward toward H, and such was its first origin. Whoever was its first inventor considered that as the triangle *ABC* coming forward raised the weight *D,* so an instrument could be constructed similar to the said triangle of some solid materia1 which, driven forward, would elevate the given weight; but then considering better how such a machine could be reduced to a much smaller and more convenient form, he took the same triangle and wound it round the cylinder *ABCD* in such a manner that the altitude *CB* of the triangle should be the height of the cylinder. Thus the ascending plane generates upon the cylinder the helical line denoted by the line *AEFGH,* which is commonly called the thread of the screw; and in this way there was created the instrument called by the Greeks *cochlea,* and by us the screw; which, turning round, comes to bear with its thread beneath the weight and easily raises it.

14. Archimede's screw.

A channel wrapped around a cylinder, is an inclined plane with a certain slope. If you now tilt the cylinder more than the slope of the channel, in some places the water will go down. By turning the screw on its axis, the water will pass from I toL to O to P ... until it pours out at the mouth H.



I should not pass over in silence here the invention of Archimedes for raising water, which is not only marvelous, but miraculous; for we shall find that the water ascends in a screw continually descending. But before going further, let us explain the use of this screw in the raising of water. Consider in the next diagram the column MIKH with the winding line ILOPQRSH round it, which is a channel through which water may run. If we piace the end I in water, tilting the screw as shown in the figure, and then turn it about the two pivots T and V, the water will go running through the channel until it finally pours out at the mouth H. Now I say that the water descends continualily in being conducted from the point I to the point H, although H is higher than I.