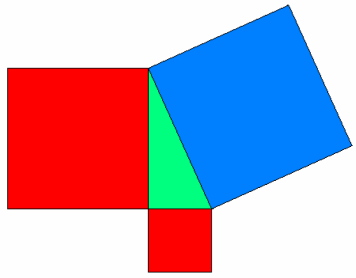
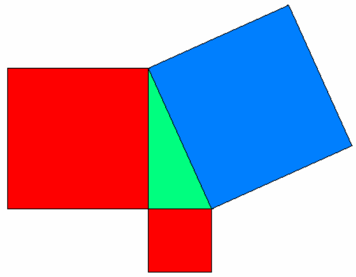
1. The theorem of Pythagoras.

In a right-angled triangle, the squares of the sides are equivalent to the square of the hypothenuse.



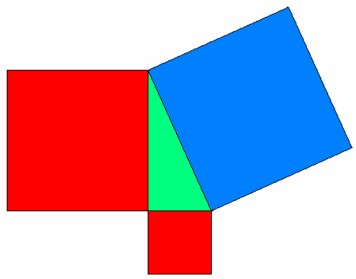
A very simple proof consists in putting in two different ways the four right-angled triangles in the large square, in such a way that the uncovered portion of the square corresponds in one position to the squares of the sides, and on the other to the square of the hypothenuse. Can you do it?

2. The theorem of Pythagoras.



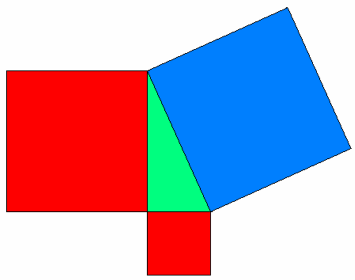
One can prove that two figures have the same area by decomposing them into the same parts. The five pieces of this puzzle can be used both to compose the squares of the sides or to reconstuct the square of the hypothenuse. Easy? Try.

3. The theorem of Pythagoras.



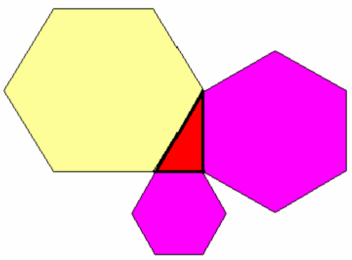
When two figures are composed of the same pieces, we say that they are equidecomposables. Two equidecomposables figures havs obviously the same area; viceversa two polygons with the same area are equidecomposables. This property, which can be proved though not easily, does not hold in three dimensions: there exist pyramids with the same volume which are not equidecomposables.

4. The theorem of Pythagoras.



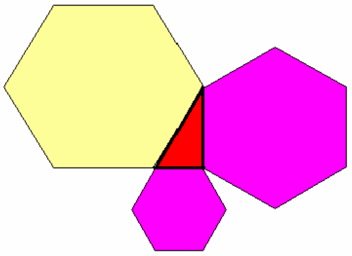
In this puzzle we have traced only the right-angled triangle, but not the squares of the sides and of the hypothenuse. Notwhistanding the fact that one of the pieces is the square of the small side and the other four pieces are equal, the solution is less easy than it might seem.

5. Similar figures.



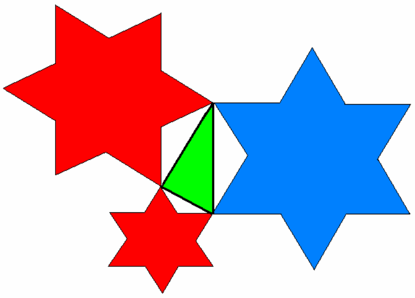
The theorem of Pythagoras remains true if instead of the squares we consider triangles, pentagons or other regular polygons. In this case, the exhagons of the sides are equivalent to the exhagon constructed on the hypothenuse.

6. Similar figures.



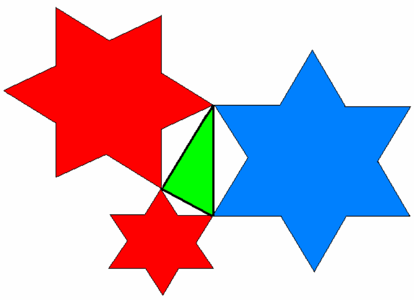
The reason of the validity of the theorem of Pythagoras for arbitrary regular polygons lies on the fact that their areas are proportional to the squares of the sides. We can verify again the theorem with a different decomposition of the exhagons.

7. Similar figures.



Actually, it is not necessary that the figures on the sides and on the hypothenuse be regular polygons; it is sufficient that they are similar figures, that is with the same form. In this case we have six-pointed stars.

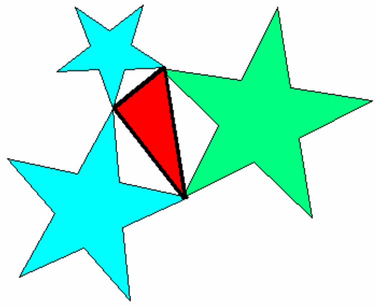
8. Similar figures.



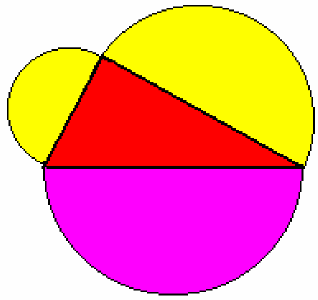
Once again, the proof lies on the fact that areas of similar figures are proportional to the squares of corresponding segments. This result is illustrated with a curious decomposition of six-pointed stars.

9. Similar figures.

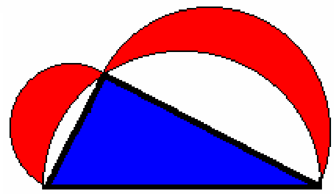
Three among these stars have the property that two of them have the same weight of the third. Can you say which ones without weighting them all, but using the theorem of Pythagoras instead? If you wish, you can verify your conjecture with the scale.



10. Lunulae.

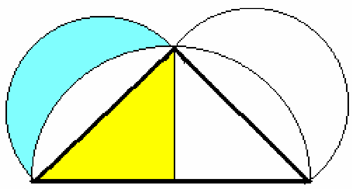


If the figures on the sides of the right-angled triangle are half circles, we have that the semicircles on the sides are equivalent to that on the hypothenuse. If now we ** this last semicircle, and we delete the parts in common with the semicircles of the sides, the remaining figures, the triangle on one side and on the other the two moon-shaped figures (whose Latin name *lunulae* means small moons), will have equal areas, as you can verify with the scale.

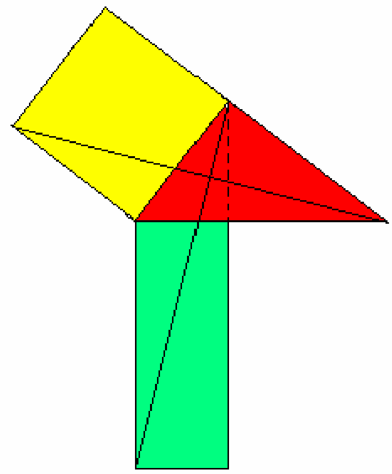


11. Lunulae.

Moreover, if the right-angled triangle has equal sides, the two lunulae will be equal too, and each of them will have the same area of half the triangle. This is the first case historically documented of a curvilinear figure being proved equal to a rectilinear one. The proof is due to Hyppocrates of Chios, who lived in the 5th century b. C. This result too can be verified with the scale.



12. The theorem of Euclid.

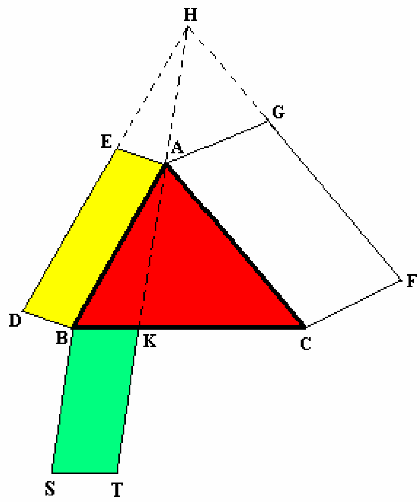


In Euclid's *Elements*, the proof of the theorem of Pythagoras goes through another result, known as Euclid's theorem:

In a right-angled triangle, the square constructed on one side is equivalent to the rectangle having as sides the hypothenuse and the projection of that side on the hypothenuse.

As always, we can verify the validity of that theorem by means of a puzzle.

13. The theorem of Pappus.



Pappus of Alexandria, a mathematician of the 4th century a. D. proved a variant of Euclid's theorem, valid also if the triangle is not right-angled and we substitute the square with an arbitrary parallelogram. The puzzle illustrates this theorem: the two parallelograms ABDE and BKST, with TK=HA, have the same area and therefore can be filled with the same pieces.

If the triangle ABC is right-angled and the parallelogram ABDE is a square, BKST is the rectangle having as sides the hypothenuse and the projection of that side on the hypothenuse, and the theorem of Pappus reduces to that of Euclid.