We will explore the asymptotic behavior as $\epsilon \to 0$ of the solution $u_\epsilon$ to the equation equation

$$\frac{\partial u_\epsilon}{\partial t} + \frac{\epsilon}{2}\Delta u_\epsilon + H_\epsilon(t, x, \omega, \nabla u_\epsilon) = 0;\ u_\epsilon(T, x) = f(x)$$

Here $H_\epsilon(t, x, \omega, p)$ is the rescaled version $H(t\epsilon^{-1}, x\epsilon^{-1}, \omega, p)$ of a stationary process

$$H(t, x, \omega, p) = H(\tau_{t,x}\omega, p)$$

with $\tau_{t,x}$ being an ergodic measure preserving action of $R^{d+1}$ on some $(\Omega, \Sigma, P)$. The basic assumption is that $H$ is convex in $p$ and satisfies some additional regularity and growth conditions.

We conclude that $u_\epsilon \to u$ which is a solution of a Hamilto-Jacobi equation

$$u_t + \bar{H}(\nabla u) = 0, u(T, x) = f(x)$$

We provide a variational formula for $\bar{H}$. The problem is related to quenched large deviation estimates for Brownian motion with a stationary random drift.